ABSTRACT. Bertrand Russell introduced several novel ideas in his 1903 *Principles of Mathematics* that he later gave up and never went back to in his subsequent work. Two of these are the related notions of denoting concepts and classes as many. In this paper we reconstruct each of these notions in the framework of conceptual realism and connect them through a logic of names that encompasses both proper and common names, and among the latter, complex as well as simple common names. Names, proper or common, and simple or complex, occur as parts of quantifier phrases, which in conceptual realism stand for referential concepts, i.e., cognitive capacities that inform our speech and mental acts with a referential nature and account for the intentionality, or directedness, of those acts. In Russell’s theory, quantifier phrases express denoting concepts (which do not include proper names). In conceptual realism, names, as well as predicates, can be nominalized and allowed to occur as “singular terms”, i.e., as arguments of predicates. Occurring as a singular term, a name denotes, if it denotes at all, a class as many, where, as in Russell’s theory, a class as many of one object is identical with that one object, and a class as many of more than one object is a plurality, i.e., a plural object that we call a group. Also, as in Russell’s theory, there is no empty class as many. When nominalized, proper names function as “singular terms” just the way they do in so-called free logic. Leśniewski’s ontology, which is also called a logic of names can be completely interpreted within this conceptualist framework, and the well-known oddities of Leśniewski’s system are shown not to be odd at all when his system is so interpreted. Finally, we show how the pluralities, or groups, of the logic of classes as many can be used as the semantic basis of plural reference and predication. We explain in this way Russell’s “fundamental doctrine upon which all rests”, i.e., “the doctrine that the subject of a proposition may be plural, and that such plural subjects are what is meant by classes [as many] which have more than one term” (Russell 1938, p. 517).

Bertrand Russell introduced several novel ideas in his 1903 *Principles of Mathematics* Russell (1938) that he later gave up and never went back to in his subsequent work. Two of these are the related notions of denoting concepts and classes as many. Russell explicitly rejected denoting concepts in his 1905 paper, “On Denoting”. Although his reasons for doing so are still a matter of some debate, they depend in part on his assumption that

* The author thanks an anonymous referee for making a number of suggestions for the improvement of this paper.
all concepts, including denoting concepts, are objects and can be denoted as such.\(^1\) Classes of any kind were later eliminated as part of Russell’s “no-classes” doctrine, according to which all mention of classes was to be contextually analyzed in terms of reference to either propositions, as in Russell’s 1905 substitution theory, or propositional functions, as in *Principia Mathematica* (Russell and Whitehead 1910). The problem with classes, as Russell and Whitehead described it in Russell and Whitehead (1910), is that if there is such an object as a class, it must be in some sense one object. Yet it is only of classes that many can be predicated. Hence, if we admit classes as objects, we must suppose that the same object can be both one and many, which seems impossible (p. 72).

Both notions are worthy of reconsideration, however, even if only in a somewhat different, alternative form in a conceptualist framework that Russell did not himself adopt. In such a framework, which we will briefly describe here, Russell’s assumption that all concepts are objects will be rejected in favor of a conceptualist counterpart to Frege’s notion of unsaturatedness, and we will reconsider the idea of a class as many somehow being both one and many.

### 1. Denoting Concepts and Classes as Many

According to Russell, a “concept denotes when, if it occurs in a proposition [as an objective truth or falsehood], the proposition is not about the concept, but about a term connected in a certain peculiar way with the concept” (Russell 1938, p. 53). In particular, denoting concepts occur in propositions the way that quantifier phrases occur in the sentences that express those propositions. Denoting concepts, in other words, are what the quantifier phrases of natural language express or stand for. Russell described only the quantifier phrases based on the six determiners ‘all’, ‘every’, ‘any’, ‘a’, ‘some’, and ‘the’, though there is no reason why phrases based on other determiners, such as ‘few’, ‘most’, ‘several’, etc., could not also be included.\(^2\) A quantifier phrase results when a determiner is applied to a common count noun, which, according to Russell, expressed a class-concept. Russell also called a quantifier phrase a denoting phrase. Thus, a “denoting phrase . . . consists always of a class-concept [word or phrase] preceded by one of the above six words or some synonym of one

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\(^1\) This occurs in his attempt to denote the meaning of a denoting phrase, which is the denoting concept expressed by that phrase. See Russell’s (1956, p. 50).

\(^2\) See Russell 1938, p. 55. We will not ourselves be concerned with these other types of determiners in this paper.