In his development of formal semantics for natural language Montague (1970a, 1973), Richard Montague modeled the meaning (Frege’s sense) of a term $A$ by its Carnap intension $\text{CI}(A)$, the function which assigns to each state $a$, specifying a “possible world”, “time” and “context of use”, the denotation $\text{den}(A)(a)$ of $A$ in that state. Now this is surely not right: among other things, it makes “there are infinitely many odd numbers” synonymous with “there are infinitely many prime numbers”, which it is not. At the other extreme, “structural” approaches to the modeling of meaning (like Russell’s propositions, Church (1946, 1974) and Cresswell (1985)) basically tell us no more than that “the sense of a complex term $A$ can be determined from the syntactic structure of $A$ and the senses or denotations of the basic constituent parts of $A$”, without explaining how this “determination” is to take place. But, to oversimplify Davidson’s eloquent criticism in Davidson (1967), Theaetetus and the property of flying do not (by themselves) amount to the meaning of “Theaetetus flies”: we would like to know just what kind of objects meanings are, and how the meaning of “Theaetetus flies” is determined by the meanings of ‘Theaetetus’ and ‘flying’.

In Moschovakis (1994) I argued that the meaning of a term $A$ can be faithfully modeled by its referential intension $\text{int}(A)$, an (abstract, idealized, not necessarily implementable) algorithm which computes the denotation of $A$. The basic technical tool in that paper was the
Formal language of recursion (FLR), for which the theory of referential intensions can be developed rigorously, and the applications to fragments of natural language were to come by “formalizing” (translating, rendering) them into FLR. Here I will develop the theory of referential intensions for the formal language $\mathcal{L}^\lambda_{ar}$, which extends the typed $\lambda$-calculus and so can accommodate (via the work of Montague) reasonably large fragments of natural language.

The claim that meanings are algorithms is a philosophical one, but this is primarily a paper in logic, not in the philosophy of language or in linguistics. Every discussion of a natural language example will start with an assertion of the form

$\text{(1) } \text{every man loves some woman}$

which will be explained and motivated when not obvious, but cannot be rigorously justified, as I will not specify with any precision the all-important rendering (or translation) operation. Some would argue that this is the most important part of the extraction of meanings from linguistic expressions, and I would agree with them. On the other hand, I think that the theory of what-happens-next proposed here may be of some value, primarily because of two reasons.

First, the modeling of meanings by referential intensions goes far beyond the imagery and analogy with computation often used to explain the relation between Frege’s sense and denotation, especially by Dummett. The explication of meaning by abstract algorithm is analogous to the “definition” of ordered pairs in axiomatic set

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4 In fact the full rendering operation is of the form

\[ \text{natural language expression + context} \xrightarrow{\text{render}} \text{formal expression + state}, \]

where the (informally understood) context determines not only the state (as we will make it precise in Section 2.2), but also which precise reading of the expression is appropriate and what formal transformations should be made (e.g., co-indexing), depending on information about “what the speaker meant”, intonation, if the expression was spoken, punctuation and capitalization, if it was written, etc. I will have nothing to say about these factors and how they determine the formal state, and so I will concentrate on the simpler, syntactical component of rendering

\[ \text{natural language expression} \xrightarrow{\text{render}} \text{formal expression} \]

for which the subsequent extraction of meaning will provide some suggestions.

5 Cf. the discussion and the references to Dummett (1978) and Evans (1982) in the introduction to Moschovakis (1994).