ITERATED BELIEF CHANGE AND THE RECOVERY AXIOM

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ABSTRACT. The axiom of recovery, while capturing a central intuition regarding belief change, has been the source of much controversy. We argue briefly against putative counterexamples to the axiom—while agreeing that some of their insight deserves to be preserved—and present additional recovery-like axioms in a framework that uses epistemic states, which encode preferences, as the object of revisions. This makes iterated revision possible and renders explicit the connection between iterated belief change and the axiom of recovery. We provide a representation theorem that connects the semantic conditions we impose on iterated revision and our additional syntactical properties. We show interesting similarities between our framework and that of Darwiche–Pearl (Artificial Intelligence 89:1–29 1997). In particular, we show that intuitions underlying the controversial (C2) postulate are captured by the recovery axiom and our recovery-like postulates (the latter can be seen as weakenings of (C2)). We present postulates for contraction, in the same spirit as the Darwiche–Pearl postulates for revision, and provide a theorem that connects our syntactic postulates with a set of semantic conditions. Lastly, we show a connection between the contraction postulates and a generalisation of the recovery axiom.

KEY WORDS: epistemic states, iterated belief change, recovery axiom

1. INTRODUCTION

A particularly simple sequence of belief change is an agent giving up and then adopting the same belief (“I believed I had money for the movies, but realised I had lost my wallet. A few minutes later, I discovered a twenty in my pocket and regained my belief that I had enough money for the movies”). The axiom of recovery in the AGM framework (Alchourron et al., 1985) places a rationality constraint on the form of such a change of beliefs. It states that expansion by a belief recovers any beliefs lost by the previous contraction by that belief. The status of the axiom of recovery has been a source of much controversy in belief revision (Fuhrmann, 1991; Hansson, 1991, 1993; Levi, 1991). There are well-known counterexamples to recovery, with the most convincing ones being Hansson’s Cleopatra and
George-the-criminal examples (Hansson, 1991, 1999). The following is a slightly amended version of the former:

I believe that ‘Cleopatra had a son’ ($\phi$) and that ‘Cleopatra had a daughter’ ($\psi$), and thus also that ‘Cleopatra had a child’ ($\phi \lor \psi$). Then I receive information that Cleopatra had no children, which makes me give up my belief in $\phi \lor \psi$. But then I am told that Cleopatra did have children, and so I add $\phi \lor \psi$. But I should not regain my belief in either $\psi$ or $\phi$ as a result.

One response to this situation is to isolate a class of belief change operators that do not satisfy recovery i.e., the so-called withdrawal operators (Makinson, 1987). We do not adopt this approach for a couple of reasons. Firstly, withdrawal operators can violate the principle of minimal change (Hansson, 1999). As an example, consider the withdrawal operator — defined as follows ($K$ is a belief set closed under a logical consequence operator $Cn$, $\alpha$ an arbitrary epistemic input): if $\alpha \notin K$, then $K - \alpha = K$, otherwise, $K - \alpha = Cn(\emptyset)$. Secondly, a fundamental intuition behind minimal contraction is the principle of core-retainment which states that if $\beta \in K$ and $\beta \notin K - \alpha$ then there is a set $K'$ such that $K' \subseteq K$ and $\alpha \notin Cn(K')$ but $\alpha \in Cn(K' \cup \{\beta\})$. It requires of an excluded sentence $\beta$ that it in some way contribute to the implication of $\alpha$ from $K$. This is only satisfied by withdrawal operators if they satisfy the recovery axiom as well. This should reinstate our faith in the recovery axiom since it is hard to find a satisfactory alternative formalization of the intuition that beliefs that do not contribute to $K$ implying $\alpha$ should be retained in $K - \alpha$.

So while the counterexamples do tickle our intuitions, it is equally the case that there is an important intuition about rational belief change that the recovery postulate captures. Indeed, the recovery postulate is best thought of as a version of the principle of minimal change: so much of the original belief state is retained on contraction that the original belief state can simply be restored on adopting the same belief. Our opinion is that even if the original postulate is rejected as being too permissive, some recovery like postulates must constrain belief revision if the principle of minimal change is to be taken seriously. Furthermore, recovery follows from other highly plausible postulates such as closure, inclusion, vacuity, success, extensionality and core-retainment (Hansson, 1999). Significantly, there is a clear and intimate connection between iterated revision and the recovery axiom: we can view the axiom as specifying the form of the iterated revision that should take place when contracting and revising by the same belief. In what follows, we make this connection clearer.