The Content of Deduction

Mark Jago

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Abstract For deductive reasoning to be justified, it must be guaranteed to preserve truth from premises to conclusion; and for it to be useful to us, it must be capable of informing us of something. How can we capture this notion of information content, whilst respecting the fact that the content of the premises, if true, already secures the truth of the conclusion? This is the problem I address here. I begin by considering and rejecting several accounts of informational content. I then develop an account on which informational contents are indeterminate in their membership. This allows there to be cases in which it is indeterminate whether a given deduction is informative. Nevertheless, on the picture I present, there are determinate cases of informative (and determinate cases of uninformative) inferences. I argue that the model I offer is the best way for an account of content to respect the meaning of the logical constants and the inference rules associated with them without collapsing into a classical picture of content, unable to account for informative deductive inferences.

Keywords Content · Information · Deduction · Inference · Epistemic scenarios

1 Introduction

Deductive reasoning is essential to philosophy, mathematics and logic. What is not so clear is how deductive reasoning conveys information to us. In ‘The
Justification of Deduction’, Dummett [4, 297] asks how deduction can be both justified and useful. If it is justified, it must be guaranteed to preserve truth from premises to conclusion. To be useful, it must inform us of something.1 But how, wonders Dummett, can the move from premises to conclusion be informative, if the former already guarantee the latter?2 The task is to capture this notion of information content whilst respecting the fact that the content of the premises, if true, already secures the truth of the conclusion. This is the aim of this paper.

According to a popular analysis, for a proposition to be informative is for it to rule out certain scenarios, or would-be possibilities.3 The proposition that it rarely rains in Cambridge is informative because it excludes possible scenarios in which it rains regularly in Cambridge. Before coming to believe that proposition, it was possible as far as I was concerned that it rains regularly in Cambridge. In coming to believe that proposition, I cease to treat such scenarios as ways the world might be, for all I know. We can think of all scenarios according to which it rarely rains in Cambridge as constituting a notion of content for ‘it rarely rains in Cambridge’ which is suitable for various epistemic purposes.4 To believe that proposition is to treat no other scenario as being a doxastic possibility; to know that proposition is to treat no other scenario as being an epistemic possibility. That content is informative for an agent iff coming to believe (or know) that proposition narrows down her doxastically (or epistemically) accessible scenarios. To be informative at all, therefore, a statement must have a non-empty content.

How should we think of the content of a valid deduction \( \Gamma \vdash A \), from premises \( \Gamma \) to conclusion \( A \), within this framework?5 We can think in terms of the differences in an agent’s belief state in the move from believing each premise in \( \Gamma \) but not believing the conclusion \( A \) to believing \( A \) as well as all premises in \( \Gamma \). Alternatively, we can think in terms of an agent discovering the incompatibility of the premises \( \Gamma \) with the negated conclusion, \( \neg A \). I’ll write

1’tFor [deduction] to be legitimate, the process of recognising the premisses as true must already have accomplished whatever is needed for the recognition of the truth of the conclusion; for it to be useful, a recognition of its truth need not actually have been accorded to the conclusion when it was accorded to the premisses’ [4, 297].

2As Dummett notes, ‘no definite contradiction stands in the way of satisfying these two requirements’; yet ‘it is a delicate matter so to describe the connection between premisses and conclusion as to display clearly the way in which both requirements are fulfilled’ [4, 297].

3Hintikka [6] gives the classic presentation of the view; Lewis [18, 19], Stalnaker [25, 26] and Chlamers [2, 3] put it to work in various ways. For recent overviews of the literature, see van Benthem and Martinez [28] and van Benthem [27].

4There are many other notions of content available. Throughout, I will take ‘content’ to mean the specific epistemic notion I’ve just described.

5I won’t make anything of the distinction between a specific procedure of deducing \( A \) from \( \Gamma \) and the statement that \( A \) can be deduced from \( \Gamma \cdot \Gamma \vdash A \). I’ll talk throughout in terms of the content of a deduction \( \Gamma \vdash A \). In saying that \( \Gamma \vdash A \) is informative, I mean that performing some derivation of \( A \) from \( \Gamma \) has the potential to be informative to some agent (or: it would be informative to an agent who begins with no declarative information at all). Thus, \( \Gamma \vdash A \) is informative only if its content is non-empty.