ON NUMERICAL APPROXIMATION USING DIFFERENTIAL EQUATIONS WITH PIECEWISE-CONSTANT ARGUMENTS

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Dedicated to the memory of Professor Miklós Farkas

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Abstract
In this paper we give a brief overview of the application of delay differential equations with piecewise constant arguments (EPCAs) for obtaining numerical approximation of delay differential equations, and we show that this method can be used for numerical approximation in a class of age-dependent population models. We also formulate an open problem for stability and oscillation of a class of linear delay equations with continuous and piecewise constant arguments.

1. Introduction

Delay differential equations (DDEs) provide a mathematical model for physical, biological systems in which the rate of change of the system depends upon their past history. The general theory of DDEs with continuous arguments has been thoroughly investigated by now, the number of the papers devoted to this area of research continues to grow very rapidly.
This paper is devoted to a generalized class of DDEs, namely delay differential equations with piecewise constant arguments (EPCAs). EPCAs include, as particular cases, impulsive DDEs and some equations of control theory, and are similar to those found in some biomedical models, hybrid control systems and numerical approximation of differential equations with discrete difference equations.

The general theory and basic results for DDEs can be found for instance in the book of Hale [20] (see also Bellman and Cooke [3], Hale and Lunel [21] and Myshkis [30]), and subsequent articles by many authors.

The study of EPCAs has been initiated by Wiener [39], [40], Cooke and Wiener [6], [7], Shah and Wiener [31]. A survey of the basic results has been given in [8], [41].

A typical EPCA is of the form

$$\dot{x}(t) = f(t, x(h(t)), x(g(t))), \quad (1.1)$$

where the argument $h$ is a continuous function and the argument $g$ has intervals of constancy. For example $g(t) = [t]$ or $g(t) = [t - n]$, where $n$ is a positive integer, and $[\cdot]$ denotes the greatest-integer function. Note that if $f(t, u, v) \equiv f_1(t, u)$, then (1.1) is a classical DDE with continuous argument. If $f(t, u, v) \equiv f_2(t, v)$, and, for instance, $g(t) = [t]$, then the solution $x$ of (1.1) is piecewise linear and at the points $t = 1, 2, \ldots$ it is equal to the solution of the discrete equation

$$y(n + 1) = y(n) + \int_n^{n+1} f_2(s, y(s)) \, ds, \quad n = 0, 1, \ldots$$

The above remark suggests that the numerical approximation of differential equations can give rise to EPCAs in a natural way, as it has been initiated by [14].

In Section 2 we explain the use of EPCAs to get a convergent numerical approximation scheme for nonlinear delay differential equations with time- and state-dependent delays, based on our earlier results [16], [17].

In Section 3 we extend these results for certain partial differential equations. We construct a numerical approximation method for a class of nonlinear age-dependent population models, and we show the convergence of the numerical scheme.

In Section 4 we discuss a linear equation which is strongly related to the stabilization of hybrid systems with feedback delays, i.e., one with continuous plant and with discrete (sampled) controller. Since some of these systems may be described by an EPCA, we formulate an open problem for investigating stability of EPCAs at the end of the paper.