INSTABILITY OF THERMAL FRACTURE UNDER THE CONDITIONS OF CONSTRAINED DEFORMATION

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We study the processes of quasistatic deformation and fracture of brittle materials under the action of rapidly varying temperature fields. As a fracture criterion, we use the condition of attainment of the critical levels of stresses. The analyses of the stressed state and crack growth are performed under the assumptions that the corresponding elements of the stress field are equal to zero on the newly formed free surfaces and that the conditions of the fracture criterion are satisfied at the ends of the crack. It is shown that the process of crack propagation is unstable for the major part of modes of thermomechanical loading: as soon as the critical stresses are attained at a certain point of the body, the crack instantaneously propagates to a critical size corresponding to a new stable state. It is shown that the mechanical overloading of a specimen can substantially weaken the effect of instability of development of the fracture zone. Examples of fracture of elastic brittle bodies are presented. We also perform the numerical analyses of the processes of initiation and propagation of cracks with regard for the plasticity of the material near its heated surface.

A well-known technological test for thermal resistance is based on the use of practically instantaneous drops of temperature leading to the fracture of brittle materials. In a simple case where temperature instantaneously rises on one side of a strip of material, it is possible to find the analytic solution of the problem, which enables one to analyze the variations of stresses in the process of gradual heating of the material. The stress fields are induced by the deformation constraints formed in the presence of nonlinear temperature fields. It can be shown that the stress fields formed in this case are such that the stresses acting at the internal points of the body are tensile (independently of the character of the stress–strain diagram of the material) and, hence, can be responsible for the violation of the integrity of the body.

The late 1960s are marked by the appearance of works devoted to the determination of conditions required for the onset of thermal fracture of some materials [1, 2]. However, the problem of subsequent development of the fracture zone remained open. In what follows, we consider one of possible scenarios of the evolution of fracture in time realized if the corresponding conditions are attained at a certain point or in a certain region.

Solution of the Problem of Heat Conduction

The process of thermal fracture is studied by analyzing, as an example, the problem of instantaneous heating of an infinite strip made of elastic brittle material.

If one of the lateral sides of an infinite free strip is instantaneously heated to a temperature $T_0$, then the temperature, strain, and stress fields depend only on the transverse coordinate $x$ and time $t$ (Fig. 1).

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We neglect the connectivity and wave effects by assuming that the time of heating is much larger than the period of propagation of waves.

The solution of the heat-conduction equation

\[ \dot{T} - k^2 T_{xx} = 0, \] (1)

where \( k^2 \) is thermal diffusivity, is sought by introducing the notion of the front of heat propagation [3] (by analogy with the proposition of Barenblatt in the theory of filtration [4]). Then

\[
\begin{align*}
T &= T_0 \left(1 - \frac{x}{l}\right)^2, & 0 \leq x \leq l, \\
T &\equiv 0, & l \leq x \leq h,
\end{align*}
\]

where \( h \) is the thickness of the strip and \( l = l(t) \) is an unknown function of time specifying the boundary of the zone of heating. The differential equation for finding \( l(t) \) is obtained by satisfying the heat-conduction equation in the integral form:

\[
\int_0^l (\dot{T} - k^2 T_{xx}) \, dx = 0, \quad (3)
\]

whence we get the following equation for \( l(t) \): \( l \cdot \dot{l} = 6k^2 \). The solution of this equation has the form

\[
l = 2k \sqrt{3t}. \quad (4)
\]