Quantum Macrostatistical Picture of Nonequilibrium Steady States

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Abstract. We apply our quantum macrostatistical treatment of irreversible processes to prove that, in nonequilibrium steady states, (a) the hydrodynamical observables execute a generalised Onsager–Machlup process and (b) the spatial correlations of these observables are generically of long range. The key assumptions behind these results are a nonequilibrium version of Onsager regression hypothesis, together with certain hypotheses of chaoticity and local equilibrium for hydrodynamical fluctuations.

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1. Introduction

It is now well appreciated that a key problem in the statistical mechanics of irreversible processes is the characterisation of nonequilibrium steady states [2, 6, 13], and a number of different approaches have been made to this problem. Some have employed rather general strategies, based, for example, on a hypothesis of Anosov dynamics [6, 13]; while others comprise treatments of concrete microscopic stochastic dynamical models [1, 4, 20].

A different approach to nonequilibrium statistical mechanics has been made by the present author in a number of works [16–18] that are centred on the hydrodynamical observables of quantum systems. This approach, like Onsager’s [11] treatment of the subject, is designed to form a bridge between the microscopic and macroscopic pictures of matter, rather than a deduction of the latter from the former. Its basic assumptions concern only very general, model-independent properties of many-particle systems, and its scope is thus intended to be complementary to that of works based on microscopic treatments of many-body problems. The results to which it has led [18] include a mathematical characterisation of local thermodynamical equilibrium and a generalisation of Onsager’s reciprocity relations to a regime where the macroscopic dynamics is nonlinear.

In this Letter, we extend the macrostatistical scheme by the introduction of a chaoticity hypothesis for the fluctuations of the nonconserved currents associated
with the locally conserved hydrodynamical observables in nonequilibrium steady states. On this basis we obtain the results that, in these states,
(a) the fluctuations of the hydrodynamical observables execute a generalised Onsager–Machlup (OM) process [12]; and
(b) the spatial correlations of these observables are generically of long range.

The latter result constitutes a mathematical generalisation of results previously proved for certain special classical stochastic models [1, 4, 20]. At a heuristic level, similar results concerning long range correlations have also been obtained from Landau’s fluctuating hydrodynamics [5, 8].

We remark here that the result (b) marks a qualitative difference between equilibrium and nonequilibrium steady states, since the hydrodynamical correlations of the former states are generically of short range*, except at critical points.

2. The Model

2.1. THE QUANTUM PICTURE

We take the model to be an \( N \)-particle quantum system, \( \Sigma \), that occupies an open, bounded, connected region, \( \Omega_N \), of a \( d \)-dimensional Euclidean space \( \mathbb{R}^d \) and is coupled at its surface to an array, \( \mathcal{R} \), of reservoirs. We assume that the particle number density, \( \nu \), of \( \Sigma \) is \( N \)-independent and that \( \Omega_N \) is the dilation by a factor \( L_N \) of a fixed, \( N \)-independent region \( \Omega \) of unit volume. Thus \( \Omega_N = L_N \Omega \equiv \{ L_N x \mid x \in \Omega \} \) and \( L_N = (N/\nu)^{1/d} \). In a standard way, we represent the observables and states of \( \Sigma \) by the self-adjoint operators and density matrices, respectively, in a separable Hilbert space \( \mathcal{H}_N \). We assume that, as has been established under rather general conditions \([14, 22]\), the composite system \( (\Sigma + \mathcal{R}) \) evolves to a unique steady state, \( \omega_N \), as \( t \to \infty \). We denote the expectation value, for this state, of an observable \( A \) by \( \langle \omega_N; A \rangle \). We shall assume that all interactions are invariant under space translations and rotations.

We assume that \( \Sigma \) has a finite set of linearly independent, extensive, conserved observables \( Q = (Q_1, \ldots, Q_n) \), which intercommute up to surface effects and are *thermodynamically complete* \([18]\) in the sense that the states corresponding to pure equilibrium phases are labelled by the expectation values of their global densities in the limit \( N \to \infty \). We denote by \( s(q) \) the equilibrium entropy density corresponding to the value \( q \) of the global density of \( Q \) in this limit.

We assume the observables \( Q \) have locally conserved, position dependent densities \( \{ \hat{q}_1(x), \ldots, \hat{q}_n(x) \} := \hat{q}(x) \), and we denote their evolutes at time \( t \), in the Heisenberg picture of the dynamics of the composite \( (\Sigma + \mathcal{R}) \), by \( \hat{q}_i^{(N)}(x, t) \equiv \hat{q}_i^{(N)}(x) = ...)

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*Our distinction between ‘long’ and ‘short’ range will be expressed in a sharp mathematical form in Section 4.*