Constant Dividend Barrier in a Risk Model with a Generalized Farlie-Gumbel-Morgenstern Copula

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Abstract In this paper, we consider the classical surplus process with a constant dividend barrier and a dependence structure between the claim amounts and the inter-claim times. We derive an integro-differential equation with boundary conditions. Its solution is expressed as the Gerber-Shiu discounted penalty function in the absence of a dividend barrier plus a linear combination of a finite number of linearly independent particular solutions to the associated homogeneous integro-differential equation. Finally, we obtain an explicit solution when the claim amounts are exponentially distributed and we investigate the effects of dependence on ruin quantities.

Keywords Compound Poisson risk model · Copula · Generalized Farlie-Gumbel-Morgenstern copulas · Constant dividend barrier · Ruin theory · Dependence models · Gerber-Shiu discounted penalty function

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1 Introduction

In the actuarial literature, risk models under a dividend strategy have been the subject of several research papers since it has been initially proposed by De Finetti (1957) for a binomial model. In the context of the classical risk model, we mention Bühlmann (1970) and Gerber (1979) who consider the problem of optimal dividend strategy in the classical compound Poisson risk model. More recently, Lin et al. (2003) carry an extensive study of the expected discounted penalty function in the...

In the classical risk model, the definition of the surplus process is based on the compound Poisson process which considers independent the interclaim times and the claim amounts (see e.g. Gerber 1979; Grandell 1991; Rolski et al. 1999). In different practical situations, this assumption may be too restrictive and more flexibility within the model is called for. In this paper, we consider the dependence structure defined with the generalized Farlie-Gumbel-Morgenstern (FGM) copula as it is proposed in Cossette et al. (2008). Other types of dependence structures for the joint distribution of the interclaim time and the claim amount have previously been considered notably in Albrecher and Boxma (2004) and Boudreault et al. (2006). Albrecher and Teugels (2006) consider a dependence structure for the interclaim times and the claim amounts based on a copula.

The objective of the present paper is to study the expected discounted penalty function in an extension with dependence to the classical risk model and assuming a constant dividend barrier strategy. In Section 2, we present the risk model with a dependence structure based on the generalized Farlie-Gumbel-Morgenstern (FGM) copula including a constant dividend barrier strategy. In Section 3, we derive the expected discounted penalty function with a constant dividend barrier as a solution to an integro-differential equation with boundary conditions. In the Appendix, the special case where the distribution of the claim amounts is exponential is considered in detail.

2 The Risk Model

For an insurance portfolio, the surplus process is $U = \{U(t), t \geq 0\}$. The surplus level at time $t$, $U(t)$, is defined by

$$U(t) = u + pt - S(t),$$

where $U(0) = u$ is the initial surplus, $p$ is the premium rate, and $S = \{S(t), t \in \mathbb{R}^+\}$ is the aggregate claim amount process. In the classical risk model, $S$ is a compound Poisson process with $S(t) = \sum_{j=1}^{N(t)} X_j$ ($\sum_{b}^{a}$ equals 0 if $b < a$) where $N = \{N(t), t \in \mathbb{R}^+\}$ is a Poisson process with parameter $\lambda$. The random variable (r.v.) $X_j$ ($j = 1, 2, \ldots$) corresponds to the amount of the $j$th claim. The time between the $(i-1)$th and the $i$th claim ($i = 2, \ldots$) is defined by the r.v. $W_i$ with $W_1$ the time of the first claim. The claim amounts $\{X_j, j \in \mathbb{N}^+\}$ form a sequence of i.i.d. r.v.’s distributed as the r.v. $X$ with probability density function (p.d.f.) $f_X$, cumulative distribution function (c.d.f.) $F_X$ and Laplace transform (L.T.) $f_X^\ast$. The interclaim times $\{W_j, j \in \mathbb{N}^+\}$ form a sequence of independent r.v.’s identically distributed as