Functional Estimation of the Random Rate of a Cox Process

Paula R. Bouzas · Ana M. Aguilera · Nuria Ruiz-Fuentes

Received: 16 November 2009 / Revised: 4 February 2010 / Accepted: 9 March 2010 / Published online: 25 March 2010
© Springer Science+Business Media, LLC 2010

Abstract The intensity of a doubly stochastic Poisson process (DSPP) is also a stochastic process whose integral is the mean process of the DSPP. From a set of sample paths of the Cox process we propose a numerical method, preserving the monotone character of the mean, to estimate the intensity on the basis of the functional PCA. A validation of the estimation method is presented by means of a simulation as well as a comparison with an alternative estimation method.

Keywords Cox process · Monotone piecewise cubic interpolation · Functional principal component analysis · Functional data analysis

AMS 2000 Subject Classification 60G51 · 60G55 · 62H25 · 46N30

1 Introduction

The Cox process (CP) or doubly stochastic Poisson process is a generalization of the Poisson process whose intensity instead of being constant (homogeneous Poisson process) or a function of time (non-homogeneous Poisson process) is also a stochastic process influenced by another external one. Due to the stochastic nature of its intensity, the CP is more flexible and realistic in order to model real phenomena.
CP was first defined by Cox (1955) and it has been deeply studied for example by Snyder and Miller (1991) or Grigoriu (1995). From a martingale point of view, CP is studied by Daley and Vere-Jones (1988), Brémaud (1981), Andersen et al. (1993), Last and Brandt (1995), among others. From all of this references we can observe that this counting process has been used in several fields like risk and insurance, medicine, signal processing, and many others.

CP is characterized by its intensity or its parametric function. The only restriction of the intensity process is that it is non negative. The parametric function of the CP is its mean and in case that it is absolutely continuous, the mean is the integral of the intensity process so that the mean is also stochastic. The mean process has non negative and nondecreasing sample paths.

Estimating the intensity process is a problem treated by many authors. For instance, Boel and Beneš (1980), Snyder and Miller (1991), Manton et al. (1999), Varini (2008), among many others, formulate several approaches using filtering methodology but it is always necessary to impose several assumptions on the intensity process. It is usual to assume that the first and second moments of the intensity are known or that it follows an explicit model. It is also more usual to approach the calculus of a linear estimation rather than non-linear even that for many applications the linearly constrained estimators are not accurate enough. In many cases, these assumptions are justified by real-world data but in many others, to establish this restrictions is just a practical way to deal with the complicated calculus of an analytic solution and computational requirements. For this reason, the development of suboptimal estimators has widely emerged.

Our work tries to deepen on how to estimate a CP without statistical assumptions on its intensity or mean processes. An attempt of relaxing the statistical assumptions was done in Bouzas et al. (2002) where it was proposed a methodology to forecast the sample paths of the CP by multivariate principal component regression in a future instant of time just from observed values.

Functional Data Analysis (FDA) models stochastic processes observed in discrete time points (as real processes can nearly always be observed) by reconstructing the functional form of their sample paths. See Ramsay and Silverman (1997) or Valderrama et al. (2000) for a deeper study. The main interest of FDA is that it is not necessary to impose a distribution to the process neither to have known moments. Bouzas et al. (2006a), proposed an estimation of the intensity process of a CP from the FDA point of view, just from observed sample paths of the CP. They were divided into subtrajectories and the intensity process was estimated in a finite set of points using a point estimator. Then, the stochastic structure of the intensity process was estimated by means of functional principal component analysis (FPCA) applied to the estimated values. In a posterior work (Bouzas et al. 2006b), it was proposed to apply FDA not to the process intensity but to the mean process of the CP and with the novelty of preserving the monotonicity property of its sample paths.

The present paper gives a new methodology of estimating the intensity process from observed trajectories of the CP, just making use of the nondecreasing monotonicity of the mean. The sketch of the present work is the following. In Section 2, once the sample paths of the CP have been observed, we obtain point estimators of the mean process at the knots of a partition of the time interval and then the functional forms of the mean sample paths are reconstructed by means of a monotone piece wise cubic interpolation (Fritsch and Carlson 1980) because of