Stochastic thermal stresses in an FGM annular disc of variable thickness with spatially random heat transfer coefficients

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Abstract The second-order statistics (i.e. mean and standard deviation) of the temperature and thermal stresses are evaluated in an axisymmetrically heated functionally graded annular disc of variable thickness with spatially random heat transfer coefficients (HTCs) on the major surfaces of the disc. This annular disc is assumed to have arbitrary variations in the HTCs and material composition along the radial direction only. The randomness in the HTCs is considered to be a random field. The stochastic temperature field is analysed by considering the annular disc to be a multilayered one with stepwise thickness variation, where each layer is assumed to have constant deterministic material properties and random HTCs. In order to evaluate the statistics, the Monte Carlo simulation method is applied to analytical solutions for the deterministic temperature and thermal stresses. The analytical solution for the thermal stresses is obtained through the use of a piecewise power function approximation for Young’s modulus. Numerical results demonstrate the effects of the magnitude of the HTC means, volume fraction distributions of the constitutive materials and thickness variation on the statistics of the temperature and thermal stresses.

Keywords Random field · Monte Carlo simulation · Functionally graded material · Thermoelasticity · Heat transfer coefficient · Applied mechanics

1 Introduction

Discs heated by their surroundings have a wide range of industrial applications, such as annular fins, turbine discs and brake disc rotors. Therefore, extensive theoretical studies on their thermal stress problems have been reported thus far. These include non-axisymmetrically heated discs [1–5], variable thickness discs [2, 6], anisotropic discs [4, 5, 7–12], elastoplastic discs [13, 14] and nonhomogeneous discs [1–3, 15, 16] such as functionally graded materials (FGMs). However, to simplify the analyses, almost all the studies were conducted with the following assumption: the major surfaces of the discs are insulated or heat is dissipated with uniform heat transfer coefficients (HTCs) throughout the surfaces.

In actual thermal environments, the HTCs of object surfaces are known to vary spatially and depend heavily on the motion of the surrounding media and the surface properties including surface asperities [17, 18]. Obviously, the effect of the spatial change in the HTCs on the thermal stress fields is large in objects with a wide heat transfer area per unit volume, such as thin strips, plates and discs. In addition, it is well known that the HTC is the most uncertain factor in an analy-
Generally, the spatial change in HTCs significantly affects the temperature distribution in objects, thereby causing a considerable change in the shape of the thermal stress distribution. Sugano [20] and Chen [21] investigated the transient thermal stresses in a rectangular plate with a unidirectional variation in the HTCs on the flat surfaces, emphasizing the importance of the location dependence of the HTCs in thermal stress analyses. Subsequently, Lee et al. [22] investigated the transient coupled thermoelasticity of an annular fin with variable HTCs along the radial direction, and Sugano et al. [23] performed the material design of functionally graded rotating discs considering the spatial change in HTCs on their surfaces. Moreover, Eraslan et al. [24, 25] developed a computational model to predict elastic-plastic stress distribution in rotating discs with a variable HTC. However, the uncertainties in the HTCs were not explicitly considered in their studies.

Only a few stochastic studies have been conducted on the heat conduction and thermal stress problems that consider the spatial or temporal randomness of HTCs. Madera [26] and Emery [27] analysed the stochastic heat conduction problem for a rectangular fin in which the HTC on the extended surface is expressed as a stochastic process and a random field, respectively. Kuznetsov [19] dealt with the stochastic heat conduction problem for an infinite strip in which the HTC is not expressed as a random field but is considered to be spatially random. However, these studies did not mention the associated thermal stresses. On the other hand, Mori et al. [28] numerically analysed the statistics of the thermal stresses in a stationary gas turbine blade heated by the surroundings via temporally random HTCs by using the stochastic finite element method. Moreover, Klevtsov et al. [29] investigated random thermal stress oscillations in steam generator tubes induced by temporally random HTCs. Fairly recently, Chiba [30] derived analytical solutions for the stochastic temperature field and thermal stresses in a heated annular disc in which the HTCs on the flat surfaces are modeled as a random field. However, these studies assumed homogeneous materials.

Currently, it appears that no literature exists on the specific topic of thermoelastic analysis of nonhomogeneous bodies involving random HTCs. If the statistics pertaining to thermal stresses in a nonhomogeneous body with an arbitrary nonhomogeneity are obtained, they will be quite useful because they can be used to estimate the reliability of FGMs with thermal stress relaxation, which have recently been developed remarkably. Since quantification is indispensable to estimate the destruction probability, extensive research has been conducted on the evaluation of stochastic thermal stress in FGMs. For example, analysis of FGM plates has been developed for random thermal boundary conditions [31], random initial temperature [32] and random material properties [33–36].

The main objective of this work is to evaluate the second-order statistics, i.e. the means and standard deviations of the temperature and thermal stresses in an axisymmetrically heated functionally graded annular disc with spatially random HTCs on the major surfaces. Since the discs used in industry have variable thickness, we consider the radial variation in the thickness. The material composition of the functionally graded disc under consideration in this study is a function of the radial coordinate only. The HTCs are assumed to vary arbitrarily along the radial direction and disperse randomly with some random amplitude around their means as a homogeneous random field. In order to evaluate the output statistics, the Monte Carlo simulation method is employed with analytical solutions for the deterministic temperature and thermal stresses. The analytical solution for temperature is obtained through Vodicka’s method [37], which is a type of integral transform method, and the analytical solution for thermal stresses is obtained through the piecewise power function approximation for Young’s modulus.

Numerical calculations are performed for functionally graded annular discs having HTC means increasing along the radial direction, which are comprised of two types of spatial variation for the material composition. The effects of the magnitude of the HTC means, volume fraction distributions of the constitutive materials and disc thickness variation on the means and standard deviations (STDs) of temperature and thermal stresses are discussed.

1The random field describing the HTCs here is homogeneous (or stationary), after subtracting the spatially varying mean from the random HTCs.