Flow and heat transfer over an unsteady stretching sheet in a micropolar fluid

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Abstract The unsteady laminar flow of an incompressible micropolar fluid over a stretching sheet is investigated. The unsteadiness in the flow and temperature fields is caused by the time-dependence of the stretching velocity and the surface temperature. Effects of the unsteadiness parameter, material parameter and Prandtl number on the flow and heat transfer characteristics are thoroughly examined.

Keywords Unsteady flow · Heat transfer · Stretching sheet · Micropolar fluid · Fluid Mechanics

Nomenclature
\[ a, b, c \] constants
\[ f \] dimensionless stream function
\[ h \] dimensionless microrotation
\[ i \] dimensionless microinertia
\[ j \] microinertia
\[ K \] material parameter
\[ m \] boundary parameter
\[ N \] microrotation
\[ \Pr \] Prandtl number
\[ S \] unsteadiness parameter
\[ T \] fluid temperature
\[ t \] time
\[ T_w \] surface temperature
\[ T_\infty \] ambient temperature
\[ u, v \] velocity components along the \( x \) and \( y \) directions, respectively
\[ U_w \] stretching velocity
\[ x, y \] Cartesian coordinates along the surface and normal to it, respectively

Greek letters
\[ \alpha \] thermal diffusivity
\[ \gamma \] spin gradient viscosity
\[ \eta \] similarity variable
\[ \theta \] dimensionless temperature
\[ \kappa \] vortex viscosity
\[ v \] kinematic viscosity
\[ \mu \] dynamic viscosity
\[ \rho \] fluid density
\[ \psi \] stream function

Subscripts
\[ w \] condition at the stretching sheet
\[ \infty \] condition at infinity

Superscript
\[ \prime \] differentiation with respect to \( \eta \)
1 Introduction

Flow of a viscous fluid past a stretching sheet is a classical problem in fluid dynamics. Crane [1] first obtained an elegant analytical solution to the boundary layer equations for the problem of steady two-dimensional flow due to a stretching surface in a quiescent incompressible fluid. Flow and heat transfer characteristics due to a stretching sheet in a stationary fluid occur in a number of industrial manufacturing processes and include both metal and polymer sheets, for example the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheets, paper production, metal spinning and drawing plastic films. The quality of the final product depends on the rate of heat transfer at the stretching surface. The problem of steady flow past a stretching surface has been extended by many authors [2–12] in various ways and very recently by Ishak [13].

All of the above mentioned studies deal with stretching sheet where the flows were assumed to be steady. Unsteady flows due to stretching sheets have received less attention; a few of them are those considered by Devi et al. [14], Andersson et al. [15], Nazar et al. [16], and very recently by Ishak et al. [17], Pal and Hiremath [18], and El-Aziz [19]. In Ref. [16], the similarity transformation introduced by Williams and Rhine [20] was used, which transforms the governing partial differential equations with three independent variables to two independent variables, which are more convenient for numerical computations.

Motivated by the above investigations, in this paper we present the characteristics of the flow and heat transfer caused by a stretching sheet in a micropolar fluid. The governing partial differential equations are transformed into ordinary ones using similarity transformation, before being solved numerically by the Keller-box method. The results obtained are then compared with those of Grubka and Bobba [3], Ali [6] and the series solution for the steady-state flow case, to support their validity.

2 Problem formulation

Consider an unsteady, two-dimensional laminar flow of an incompressible micropolar fluid over a stretching sheet, as shown in Fig. 1. At time $t = 0$, the sheet is impulsively stretched with velocity $U_w(x, t)$ along the $x$-axis, keeping the origin fixed in the fluid of ambient temperature $T_\infty$. The stationary Cartesian coordinate system has its origin located at the leading edge of the sheet with the positive $x$-axis extending along the sheet, while the $y$-axis is measured normal to the surface of the sheet. The boundary layer equations may be written as [21, 22]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left( \frac{\mu + \kappa}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \kappa \frac{\partial N}{\partial y},$$

$$\rho j \left( \frac{\partial N}{\partial t} + u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} \right) = \frac{\partial}{\partial y} \left( \gamma \frac{\partial N}{\partial y} \right) - \kappa \left( 2N + \frac{\partial u}{\partial y} \right),$$

$$\frac{\partial j}{\partial t} + u \frac{\partial j}{\partial x} + v \frac{\partial j}{\partial y} = 0,$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2},$$

where $u$ and $v$ are the velocity components in the $x$- and $y$-directions, respectively, $T$ is the fluid temperature in the boundary layer, $N$ is the microrotation or angular velocity, and $j$, $\gamma$, $\mu$, $\kappa$, $\rho$, and $\alpha$ are the microrotation, spin gradient viscosity, dynamic viscosity, vortex viscosity, fluid density and thermal diffusivity, respectively. It is assumed that the stretching velocity $U_w(x, t)$ and the surface temperature $T_w(x, t)$ are of the form

$$U_w(x, t) = \frac{ax}{1 - ct}, \quad T_w(x, t) = T_\infty + \frac{bx}{1 - ct},$$

where $a$, $b$ and $c$ are constants with $a > 0$, $b \geq 0$ and $c \geq 0$ (with $ct < 1$), and both $a$ and $c$ have dimension $^{-1}$. It should be noticed that at $t = 0$ (initial motion), (1)–(3) describes the steady flow over a stretching surface. This particular form of $U_w(x, t)$