MATHEMATICAL MODEL OF THE MOVEMENT OF LAYERS OF METAL DURING THE ROTARY ROLLING OF SEMIFINISHED PRODUCTS MADE OF HYPEREUTECTIC SILUMINS

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A mathematical model has been constructed to describe the kinematics of the movement of metal on the surface of semifinished products made of hypereutectic silumin alloys as they undergo rotary rolling. The model is based on the dependence of the pitch of the helical lines of metal flow on the deformation $\varepsilon$ of the product and the feed angle $\alpha$. The pitch is minimal when $\varepsilon = 50–55\%$, regardless of the feed angle $\alpha$. The degree of deformation $\varepsilon = 50–55\%$ is optimum for ensuring that the unit number of cycles in which the semifinished product is loaded by the work rolls is such as to produce a fine-grained structure and maximize the ductility characteristics of hypereutectic silumins.

Rotary rolling is a process that involves subjecting a cylindrical semifinished product to strictly axisymmetric deformation. In light of this, it can be assumed that the concentric layers of metal which comprise the semifinished product do not move in the radial direction during the rolling operation, i.e., the layers are in the same position after rolling that they were before rolling. This assumption is made because there are no physical reasons that would cause the outer, middle, and central layers of the product to move and become intermixed with one another.

The above assumption is strongly validated by the results of experiments that involved the rotary rolling of a semifinished product with long pins embedded in it [1]: the pins farthest from the geometric axis remained the pins farthest from that axis after rolling, while the pins closest to the axis also remained so after the rolling operation. However, the same experiments did clearly show that all the layers of the rolled specimen undergo helical (helicoidal) deformation. In other words, it was found that the layers move in the tangential direction as they undergo twisting and move in the longitudinal direction as they undergo elongation.

In addition, while being twisted and stretched during rotary rolling, all the concentric layers of the metal move radially in the direction of the geometric axis of the semifinished product. This movement ensures reduction of the semifinished product as a whole. Thus, all the layers of the specimen undergo complex volumetric deformation.

We will examine the kinematics of the movement of an outer layer of a specimen made of hypereutectic silumin alloy 01390. This is the case that is most accessible for visual examination, since it is relatively easy to use traces from the flow of the surface layer of metal to determine one of the parameters that is key to describing the kinematics of rotary rolling – the pitch $S$ of the helical line followed by the metal as it moves over the surface of the specimen [1]. For the deformation zone, this parameter is determined from the following formula [2]:

$$S_x = \pi d \frac{F_0}{F_x} \frac{D_0}{D_x} \frac{\eta_{ax}}{\eta_l} \tan \alpha,$$
where $S_x$ is the pitch of the helical line followed by the metal in its motion over the surface of the semifinished product in the section $x$ of the deformation zone; $d_x$ is the diameter of the semifinished product in the given section $x$; $F_o, F_x, D_o$ and $D_x$ are the cross-sectional areas of the semifinished product and the roll in the outlet section (the exit from the deformation zone) and in section $x$, respectively; $\eta_{ax}$ and $\eta_t$ are the coefficients of axial and tangential slip, respectively; $\alpha$ is the feed angle.

It is evident that the pitch of the helix is smaller at the beginning of deformation than in the outlet section of the deformation zone. In fact, the helical lines that remain on the surface of the rolled specimen are formed in the outlet section. In this case, the pitch of each helix $S_{sfc}$ is determined from the formula

$$S_{sfc} = \pi d_{r_b} \frac{\eta_{ax}}{\eta_t} \tan \alpha, \quad (1)$$

where $d_{r_b}$ is the diameter of the rolled bar.

The problem of determining the pitch $S_{sfc}$ reduces to determining the ratio $\eta_{ax}/\eta_t$. This ratio can vary within a broad range of values and depends on the following factors: the material of the semifinished product and the temperature to which it is heated; the degree of deformation $\varepsilon$ of the semifinished product; the feed angle $\alpha$; the reeling angle $\beta$; the scheme used for rotary rolling, etc.

In our case, optimum values were found experimentally for some of the variables and were used as the initial data, i.e., as fixed parameters: three-roll rolling scheme [3]; temperature to which the semifinished products are heated 470–480°C [4]; reeling angle $\beta = 4^\circ$; material of the semifinished products – silumin alloys 01390, 01391, and 01392. Thus, the variable parameters that need to be evaluated in order to determine $S_{sfc}$ are the degree of deformation $\varepsilon$ and the feed angle $\alpha$.

Preliminary tests showed that the function $\eta_{ax}/\eta_t = f_1(\alpha)$ is close to linear for $\varepsilon = \text{const}$, so that it is approximated well by the relation

$$\eta_{ax}/\eta_t = 1 - k\alpha, \quad (2)$$

where $k$ is a proportionality factor that depends on the deformation $\varepsilon$.

The experimental data showed that $k = f_2(\varepsilon)$ can be approximated by the equation [5]

$$k = C - B(\varepsilon - A)^2, \quad (3)$$

where $A$, $B$, and $C$ are constants determined on the basis of experimental data (Table 1) and the average values $k_{av}$:

- for $\varepsilon = 0.35$ $k_{av} = (0.0208 + 0.0189)/2 = 0.0198$;
- for $\varepsilon = 0.49$ $k_{av} = (0.0283 + 0.0278)/2 = 0.0281$;
- for $\varepsilon = 0.71$ $k_{av} = (0.0233 + 0.0222)/2 = 0.0228$.

We take the three values of $k_{av}$ and $\varepsilon$ and construct a system of quadratic equations, then finding the constants $A = 0.548$, $B = 0.2316$, and $C = 0.02888$. Using these constants and Eq. (3), we obtain a formula to determine $k$:

$$k = 0.02888 - 0.2316(\varepsilon - 0.548)^2. \quad (4)$$

After inserting the value of $k$ from (4) into Eq. (2), we find the ratio $\eta_{ax}/\eta_t$:

$$\eta_{ax}/\eta_t = 1 - \alpha[0.02888 - 0.2316(\varepsilon - 0.548)^2]. \quad (5)$$

Finally, inserting Eq. (5) into Eq. (1), we obtain an equation for the pitch of the helix followed by the metal on the surface of the rolled bar:

$$S_{sfc} = \pi d_{r_b} \tan \alpha \{1 - \alpha[0.02888 - 0.2316(\varepsilon - 0.548)^2]\}. \quad (6)$$