A method is considered for calculating the basic error of a measuring instrument from test data by the employment of structural and parametric identification of probability distributions on the criterion for maximum reproducibility.

Key words: unified measurements, verification, basic error, structural and parametric identification, probability distribution, statistical distribution.

Unified measurements involve the assumption that the errors in the measurements are known and that with a given probability do not exceed established limits [1]. The error of a measurement is considered as known if the probability distribution is known for the possible values and the parameters of that distribution [2], and within those limits there is a proportion of the distribution corresponding to the fiducial probability in the test scheme for that form of measurement: 0.90, 0.95, or 0.99 [3].

Here I consider identifying the probability distribution for a basic error of a measuring instrument (MI) in order to demonstrate the parameters in the test methods and confirm that they correspond to the [4] standard.

Observability of MI Error Components. The above limits and proportions of probability distribution for the error values correspond to an important concept in mathematical statistics: the \( P, \gamma \) tolerance interval. This interval with fiducial probability \( P \) contains a proportion \( \gamma \) of the probability distribution [5]. However, the distributions are essentially related to the observability of the corresponding measured and calculated quantities.

Observability in metrology from the viewpoint of this very important physics concept [6] involves the errors of measurements containing not only an observed random component, which makes itself felt in repeated measurements under unal-
tered conditions as random changes in the MI readings, but also contains an unobserved component [7] called the residual systematic error (RSE). It becomes accessible to observation, measurement, or determination at higher levels of the test schemes in relation to working standards and in the analysis of measurements in a scheme for cross observation of lack-of-fit errors in the mathematical model for the measurement object [8].

The mathematical model for the basic error of an MI is the distribution of the sum of the observed random component $\Xi_*$ with probability distribution (PD) of the form $\sim F_*(\Theta_*; \xi)$ and the residual systematic component $\Psi_R$ unobserved on verification [9], which in test schemes according to [10] is assigned a uniform distribution with dispersal parameter $\theta_{R2} = \theta_{R0}$ as the limit to the permissible values of the RSE:

$$f_{\Delta}(\delta) = \frac{F_*(\Theta_{R}; \delta + \theta_{R0}) - F_*(\Theta_{R}; \delta - \theta_{R0})}{2\theta_{R0}}. \tag{1}$$

Formally speaking, (1) in the form of the central difference of PD of the form $\sim$ shows the uniqueness of the characteristic function $\phi_*(t)$ of the probability density distribution PDD $f_*(\xi)$ for $h \to 0$ [11]. The left-hand side in this expression is the PDD for any $h > 0$, and if $h = \theta_{R2}$, it is considered as the dispersal parameter of the uniform ($\sim R$) distribution with position parameter $\theta_{R1} = 0$.

The uncertainty parameter of the (1) PDD is the sum of the dimensional, parametric, and structural components [7] and is represented as [2]

$$\theta_{R0} = \theta_{\max} + t_{\alpha, N-1} s(\hat{\theta}_*_1) + D_*/f_*(\hat{\theta}_*_1), \tag{2}$$

where $\theta_{\max}$ is the limit to the permissible values of the RSE for the measuring instrument, $t_{\alpha, N-1}$ is the quantile of the Student’s distribution with $N - 1$ degrees of freedom at level $\alpha$ in accordance with the established test scheme for the fiducial probability; $N$ is the number of measurements; $s(\hat{\theta}_*_1)$ is the standard deviation in the maximum likelihood estimator for the position parameter $\theta*_1$; $D_*$ is the Kolmogorov distance between the statistical distribution function (SDF) for the random component of the measuring instrument error $F_*(\xi)$ and the equivalent PD $F_*(\xi)$; and $f_*(\hat{\theta}_*_1)$ is the modal value of the PDD [2].

It may be difficult to use (2) in applied measurements by the lack of data on the RSE, since for many MI one normalized [12] the sum of the random error and residual systematic error in distinction from componentwise standardization of the metrological characteristics in accordance with [13]. Therefore, the calculations must incorporate the parameters of the methods of checking these MI.

The series expansion of (1) for a Gauss distribution ($\sim G$) with $\theta_{G2} < 0.173\theta_{GR0}$ gives a standard relation [14]:

$$f_{GR}(\delta) = \left[\frac{\sqrt{2\pi(\theta_{G2}^2 + \theta_{GR0}^2 / 3)}}{\sqrt{2\pi(\theta_{G2}^2 + \theta_{GR0}^2 / 3)}}\right]^{-1} \exp \left[-\frac{(\delta - \theta_{GR1})^2}{2(\theta_{G2}^2 + \theta_{GR0}^2 / 3)}\right].$$

The Jordan distribution is preferable here if of course one can resolve the problem of the shape parameter.

Also, in connection with (1) one needs to pay attention to the inaccurate use in many methods of checking MI of the term “actual value of a physical quantity” to readings of the working standard, i.e., when its error cannot be neglected.

Then in checking an MI, one should not restrict consideration to the random component observed in the form of an a posteriori difference in readings between the instrument and the working standard in the determination of the degree of correspondence between the basic error and the region of permissible values [15]. At the same time, incorporating the RSE should not be restricted to considering the error characteristics of the working standards, which appear in checking as a priori data. For probability distributions in that category, one also has errors of lack of fit, which arise from the errors in the statistical estimation of the parameters and errors in the choice of the probability distribution.