THERMAL MEASUREMENTS

THE PRINCIPLES OF DATA PROCESSING BASED ON THE NUMERICAL SOLUTION OF THE NONLINEAR HEAT-CONDUCTION PROBLEM
Pt. 2. THE COEFFICIENT FUNCTIONAL OPTIMIZATION PROBLEM IN A VARIATIONAL FORMULATION

I. N. Ishchuk

The solution of the coefficient heat-conduction problem when a linear pulsed heat source acts in the plane of contact of two semibounded bodies, obtained using variational-iterational calculus methods, is presented. The results of tests on a number of heat-insulating materials are given.

Key words: thermal properties, variational calculus, difference schemes, mathematical modeling.

We will present the solution of the direct heat-conduction problem obtained in [1] in operator form as $\omega = Ru$. The direct problem consists of calculating $\omega$ for known $u$, where $R$ is the operator of the linear mapping of the function $u$ into the function $\omega$. Suppose the function $\omega$ is known with a certain error $\delta$, i.e., of the “actual” function $\tilde{\omega}$ we only know that $|\omega - \tilde{\omega}| \leq \delta$. It is required to determine the function $u$ in order that $\|Ru - \omega\| \rightarrow \min$.

Using the theory of the regularization of ill-posed problems, we will consider a certain construction of the solution of the inverse heat-conduction problem [2, 3]. It is required to obtain the function $u$ of the solution of the problem in the form

$$
\min_{u(\cdot)} \left\{ \|Ru - \omega\|^2 + \varepsilon \|du / dx\|^2 \right\};
$$

$$
\min_{u(\cdot)} \|Ru - \omega\|, \quad \text{for} \quad \|du / dx\| \leq A
$$

using an additional estimate of the deviation of $\omega$ from $\tilde{\omega}$ as $\rho(\omega, \tilde{\omega}) \leq \delta$, where $\{\varepsilon, A\} \rightarrow \xi$ are regularization parameters.

We will formally write the solution of this problem as

$$
u = R_\xi^{-1} \omega,$$

where $R_\xi^{-1}$ is the regularized inverse operator $R^{-1}$, $R^{-1}$ is unbounded, while $R_\xi^{-1}$, which is very important, approximates $R^{-1}$ with good accuracy. The accuracy of the approximation depends on the method of regularization and the parameter $\xi$. In addition to the fact that $R_\xi^{-1}$ is unbounded, the norm $\|R_\xi^{-1}\|$ is an important characteristic, on which the relation between the accuracies of the specification of $\omega$ and the answer $u$ depends.

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Tambov Higher Military Aviation Engineering College of Radio-Electronics (Military Institute); e-mail: boerby@rambler.ru. Translated from Izmeritel’naya Tekhnika No. 2, pp. 48–50, February, 2008. Original article submitted June 4, 2007.
A subjective method of solving the problem has been used for a long time in addition to the variational method considered. It consists of the fact that a specialist-interpreter chooses a function which satisfies a specific condition, in particular, the discrepancy criterion and the constraints. These properties are often not formulated explicitly, but the interpreter knows what functions can be used in this problem and which ones cannot [4].

We will consider the solution of the inverse problem in two stages: the identification of the coefficients of the discrete mathematical model, and on the basis of these, the identification of the thermal properties of the heat-insulating materials being tested.

The coefficients of the thermal model were identified from the results of experimental investigations of certain heat-insulating materials. We will consider the variational formulation of the functional optimization of the problem of identifying the coefficients $K_i$ and $K_{j\lambda}$ of a piecewise-linear finite function (see formula (3) in [1]), where $a$ is the thermal diffusivity, $\lambda$ is the thermal conductivity, $j = 1, 3$, and the parameters $q$, $\alpha$, and $\beta$ are the implicit mathematical model of the quantity of heat, and the coefficients of the heat losses and of the mathematical model, respectively.

Using gradient iteration methods, we minimize the smoothing mean-weighted discrepancy functional with respect to three standard materials: 1) polyurethane foam (PUF) – $a_1 = 2.9 \cdot 10^{-7}$ m$^2$/sec and $\lambda_1 = 0.31$ W/(m·K), 2) low-molecular

![Fig. 1. Results of a measurement and modeling of the excess temperature for different tested materials: 1) PUF; 2) SCCC; 3) LMHRSR(Z).](image1)

![Fig. 2. Results of a measurement and modeling of the excess temperature using the identified thermal properties of LMHRSR(S): $a_1 = 4.2 \cdot 10^{-7}$ m$^2$/sec and $\lambda_1 = 0.192$ W/(m·K).](image2)