THE METHOD OF THREE-POINT BENDING IN TESTING THIN
HIGH-STRENGTH REINFORCED PLASTICS AT LARGE
DEFLECTIONS

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Keywords: advanced composites, test methods, thin bar, analytical model, three-point bending, large deflec-
tions, tension, strength

A method is presented for determining the flexural strength of unidirectional composites from three-point
bending tests at large deflections. An analytical model is proposed for calculating the flexural stress in testing
thin bars in the case of large deflections. The model takes into account the changes in the support reactions at
bar ends and in the span of the bar caused by its deflection. In the model offered, the influence of transverse
shear and the friction at supports are neglected. The problem is solved in elliptic integrals of the first and sec-
ond kind. The results obtained are compared with experimental tension data. The method elaborated for cal-
culating flexural stresses has an obvious advantage over the conventional engineering procedure, because the
calculation accuracy of the stresses increases considerably in the case of large deflections.

Introduction

The tensile strength is one of the most important characteristics of composite materials [1, 2]. Advanced materials
made with unidirectional high-strength (to 5-7 GPa) carbon fibers, manufactured by pultrusion, possess a high tensile strength
[3, 4]. The strength of high-strength and high-modulus unidirectional carbon-fiber-reinforced plastics (CFRP) is usually deter-
dined on strip specimens with a constant transverse cross section. However, since the specimens often fail near clamps due to
the concentration of stresses in the force transfer zone [5-7], the results obtained are unreliable.

An alternative to the tensile tests is the tests of specimens in bending. The three- and four-point loading schemes, elab-
orated for testing isotropic materials, are usually applied to modern composite materials too [1, 2]. However, these methods
also have considerable shortcomings. The main imperfection consists in the fact that the specimens frequently fail under the
point force from a stress concentration. The magnitude of the stress concentration depends on the point force: the greater the
force, the higher the stress concentration. This force can be reduced either by increasing the distance between supports or by re-
ducing the thickness of the specimen. However, in this case, the specimens will fail at large deflections.

For determining the stress state in three-point bending, the simplified linear engineering theory of beams is commonly
used [1]. The deflection of the bar is assumed small compared with the span, which makes it possible, in the differential equa-
tion of the bent axis of the bar, to neglect the squared first derivative in the equation of bending moment and to simplify the pro-
cessing of experimental data considerably. In the case of large deflections, the neglect of the squared first derivative in the
equation of bending moment leads to an error in determining the stress state and, consequently, to unreliable results for the strength. Thus, the engineering theory of bending cannot be used at large deflections of bars.

In the present study, we consider a possibility of applying the method for determining the flexural strength from three-point bend tests of specimens in the case of large deflections. Analytical dependences are derived for calculating the stress state at large deflections. The values of flexural strength calculated by the new method and based on the technical theory of bending are compared between themselves and with those obtained from tensile tests.

Procedure for Calculating the Flexural Stress of a Thin Bar in Three-Point Bending

The scheme of testing specimens in three-point bending is shown in Fig. 1. The simplest type of supports — a prism (with a radius of rounding less than 0.2 mm) — is assumed. In solving the problem, we neglect the effect of transverse shears, i.e., we use the hypothesis of undeformed cross sections. We assume that the bar is flexible and its stresses, even at a strong curving, do not exceed the limit of proportionality (and the limit of elasticity), i.e., all strains are very small. For a flexible bar, the expression of bending moment must necessarily take into account displacements arising in the bar, which are neglected in the engineering theory of bending. Therefore, all support reactions (forces and moments) considerably depend on the values of displacements in bending. The support reactions \( P \) at large angular displacements from bending under the action of a force \( Q \) are determined as

\[
P = \frac{Q}{2\cos \vartheta_1},
\]

where \( \vartheta_1 \) is the angle of deviation of the force \( P \) from the vertical, i.e., the rotation angle of the bar end. In addition, we must also take into account the change in length of the bent part of the bar due to its slipping between the prisms during the bending process. In solving the problem, the friction on the supports is neglected.

We will use the approach described in [8]. Let us consider a bent bar of width \( w \) and thickness \( t \) with a rectangular transverse cross section.

The maximum flexural stresses at the instant of failure of the bar are found from the formula

\[
\sigma_b = \frac{M_{\text{max}}}{W}, \quad (1)
\]

where \( W = wt^2/6 \) is the section modulus of the bar in the plane of bending, and \( M_{\text{max}} \) is the maximum bending moment. In the linear engineering theory of bending, \( M_{\text{max}} \) is given by

\[
M_{\text{max}} = \frac{Q}{2} a. \quad (2)
\]

For an exact determination of the bending moment, we set up the corresponding equation with account of displacements arising in the thin bar. For this purpose, we divide the elastic line of the bar by the point of application of the force \( Q \) into two equal symmetric parts and consider one of them (Fig. 2). The bending occurs under the action of the point forces \( P \) and the bending moment \( M^1_0 \).

Let us take two coordinate systems: \( xy \), oriented along the tangent and the normal to the elastic line at the point \( O \), and \( x^1 y^1 \), oriented along the force \( P \). We assume the following designations: \( \delta \) is the angle between the force \( P \) and the \( x \) axis; \( \zeta \) is the current angle between the tangent to the elastic line and the \( x^1 \) axis; \( \vartheta \) is the slope angle of the tangent to the elastic line in the \( xy \) axes; \( ds \) is the arc element of the bar. The angles introduced are connected by the relation \( \zeta = \vartheta + \delta \).

At large deflections, the bending moment \( M^1_0 \) at the midpoint of the bar, according to [8], is