A DYNAMICAL (TIME-HARMONIC) AXISYMMETRIC STRESS FIELD IN A FINITELY PRESTRETCHED MULTILAYERED SLAB ON A RIGID FOUNDATION

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Within the framework of a piecewise homogeneous body model, with the use of the three-dimensional linearized theory of elastic waves in initially stressed bodies, the dynamical (time-harmonic) axisymmetric stress field in a finitely prestretched multilayered slab resting on a rigid foundation is studied. It is assumed that the slab consists of two-layer packets. The elasticity of layer materials is described by the Treloar potential. It is assumed that the material of the lower layer in the packets is more rigid than that of the upper one. Numerical results are presented for the cases where the number of layers (packets) in the slab is 2 (1), 4 (2), or 6 (3). These results concern the normal stresses acting on the interface between the layers of the first, upper packet and on the interface between the first and second packets. The influence of the number, prestretch level, and thickness of the layers on relationships between the stresses and the frequency of the external force is analyzed.

1. Introduction

Investigations into the influence of initial stresses on the dynamical stress state in a multilayered medium within the framework of a piecewise homogeneous body model, with the use of the three-dimensional linearized theory of elastic waves in initially stressed Bodies (TLTEWISB), are of great, both theoretical and practical, significance. Up to now, only a few studies are known in this field, which are presented in [1-5], with some others listed therein.

In these investigations, it was assumed that the region occupied by the body is semi-infinite. Therefore, the results obtained in [1-5] cannot be applied, for example, to the cases where the aforementioned dynamical stress field is studied for a layered material resting on a rigid foundation. Also, these results are inapplicable to structural elements whose basic material is covered with layered ones. Because of this, the investigations carried out in [1-5] were developed in [6] for a two-layer slab resting on a rigid foundation. However, in many cases, the covering slab may consist of more than two layers. Therefore, in the present paper, the results obtained in [6] are extended to slabs containing any number of finitely prestretched layers. As in [6], it is assumed that a time-harmonic normal point force acts on the face plane of the upper layer of the slab, and the axisymmetric stress state in this slab is studied. Furthermore, it is assumed that the layers of the slab are finitely prestretched in the radial di-
reactions, and the slab thickness increases with number of layers in it. We suppose that the layer materials are incompressible neo-Hookean materials, and the stress–strain relations for them are given through the Treloar potential. The investigations are carried out within the framework of a piecewise homogeneous body model by the use of TLTEWISB.

It should be noted that the related recent investigations on the wave propagation problems are presented in [7-11].

2. Formulation of the Problem and the Solution Procedure

We consider a multilayered slab resting on a rigid foundation (Fig. 1). In the natural state, the thicknesses of the layers are \(h_1, h_2, \ldots, h_N\) (Fig.1), and they are related to Lagrangian coordinates in the Cartesian \(Oy_1y_2y_3\) and cylindrical \(O\theta y_3\) reference frames. The layers are infinite in the radial directions. Before compounding with one another and with the rigid foundation, the layers are stretched separately along the radial directions, as a result of which a homogeneous axisymmetric initial strain state appears in each of them.

With the initial state of the layers, we associate Lagrangian cylindrical \(O\theta'\theta y_3'\) and Cartesian \(O'y_1'y_2'y_3'\) systems of coordinates. The layer materials are incompressible neo-Hookean, and the quantities related to a \(k\)th \((1 \leq k \leq N)\) layer will be labeled with the superscript \((k)\). In addition, the quantities related to the initial state will have the superscript 0. Thus, according to the above-stated, the initial state of the layers can be determined as follows:

\[
u^{(k),0}_m = (\lambda_m^{(k)} - 1)y_m, \quad \lambda_1^{(k)} = \lambda_2^{(k)} \neq \lambda_3^{(k)}, \quad \lambda_m^{(k)} = \text{const},
\]

\[
\lambda_1^{(k)}\lambda_2^{(k)}\lambda_3^{(k)} = 1, \quad m = 1, 2, 3, \quad k = 1, 2, \ldots, N,
\]

where \(u^{(k),0}_m\) is the displacement and \(\lambda_m^{(k)}\) is the elongation along the \(Oy_m\) axis. Let us introduce the designation

\[
\hat{\lambda}_1^{(k)} = \hat{\lambda}_2^{(k)} = \lambda^{(k)}, \quad \hat{\lambda}_3^{(k)} = (\lambda^{(k)})^{-2}.
\]

It follows from (1) that

\[
y_i' = \hat{\lambda}_1^{(k)}y_i, \quad r' = \hat{\lambda}_3^{(k)}r, \quad h'_k = (\lambda^{(k)})^{-2} h_k.
\]

In what follows, the quantities related to the system of coordinates associated with the initial state, i.e., with \(O'y_1'y_2'y_3'\), will be primed.

According to [6,7], we write the basic relations of TLTEWISB for an incompressible body in an axisymmetric stress-strain state. These relations are satisfied in each layer, because we use a piecewise homogeneous body model.

The equations of motion in displacements are

\[
\begin{align*}
\frac{\partial}{\partial t} \lambda^{(k)} & = 0, \\
\frac{\partial}{\partial t} \lambda^{(k)} y_i' & = \frac{\partial}{\partial y_i'} \left( \frac{\partial \Pi^{(k)}}{\partial \lambda^{(k)} y_i'} \right), \\
\frac{\partial}{\partial t} r' & = \frac{\partial}{\partial r'} \left( \frac{\partial \Pi^{(k)}}{\partial r'} \right), \\
\frac{\partial}{\partial t} h'_k & = \frac{\partial}{\partial h'_k} \left( \frac{\partial \Pi^{(k)}}{\partial h'_k} \right).
\end{align*}
\]