DISTRIBUTED ZENER–STROH CRACKS GIVE A GRIFFITH CRACK, AND THEIR DIPOLE IS THE BUECKNER WEIGHT FUNCTION

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Dedicated to Professor Vitauts Tamuzs on his seventieth birthday

The Zener–Stroh crack is a crack that is dislocated in the sense of Volterra. It is shown in this article that, by distributing on a line segment Zener–Stroh cracks of constant density and adding a big Zener–Stroh crack to cancel the total Burgers vector, the resulting stress field is that of the Griffith crack. Moreover, the dipole of a Zener–Stroh crack corresponds to the Bueckner weight function.

1. Introduction

The Griffith crack is familiar to everybody; the Zener–Stroh crack is much less so. To make this article readable, both must be reviewed briefly from a certain point of view. For simplicity only the infinitely extended material is considered and the mathematics woven around, what might be called mode I for both. If the connection between the two cracks were to develop eventually into something significant physically or mathematically, the extension to bodies with finite boundaries and other modes could be done easily.

2. The Griffith Crack

If the famous Griffith crack in a tension field (see Fig. 1) is simulated by distributing a climb array of edge dislocations with a density \( B(x) \), the governing integral equation is

\[
\frac{1}{\pi} \int_{-l}^{l} \frac{B(\xi)d\xi}{\xi-x} = \frac{T(\kappa + 1)}{2\mu}, \quad |x| < l,
\]

where \( \mu \) denotes the shear modulus and, with \( \nu \) as Poisson's ratio, \( \kappa = 3 - 4\nu \) for plane strain. Since the displacement discontinuity is restricted to the interval \( |x| < l \) occupied by the crack, the total amount of dislocations inside the crack must vanish, and the integral equation must be solved imposing the side condition

\[
\int_{-l}^{l} B(x)dx = 0
\]
The solution of the integral equation is then

\[
B(x) = \frac{T(\kappa + 1)}{2\mu} \frac{x}{(l^2 - x^2)^{1/2}}, \quad |x| < l.
\]  

(3)

The gap between the crack faces is

\[
g(x) = u_y(x, 0+) - u_y(x, 0-) = -\int_{-l}^{x} B(\xi) d\xi, \quad |x| < l.
\]  

(4)

and substituting from (3)

\[
g(x) = \frac{T(\kappa + 1)}{2\mu} (l^2 - x^2)^{1/2}, \quad |x| < l.
\]  

(5)

The normal tractions on the plane of the crack are

\[
N(x) = \sigma_{yy}(x, 0) = T - \frac{2\mu}{\pi(\kappa + 1)} \int_{-l}^{l} \frac{B(\xi) d\xi}{\xi - x}, \quad |x| < \infty.
\]  

(6)

and again substituting (3) gives

\[
N(x) = T - \frac{|x| H(|x| - l)}{(x^2 - l^2)^{1/2}}, \quad |x| < \infty.
\]  

(7)

3. The Zener–Stroh Crack

The term Zener–Stroh crack was recently introduced by Weertman [1] to give credit to Zener, who proposed in 1947 that edge dislocations piling up against an obstacle could coalesce into a crack nucleus, and to Stroh, who did the required calculations in 1954. The governing integral equation for the Zener–Stroh crack is

Fig. 1. Griffith (a) and Zener–Stroh (b) cracks.