EQUAL-STRESSED REINFORCEMENT
OF METAL-COMPOSITE PLATES WITH FIBERS
OF CONSTANT CROSS SECTION IN STEADY-STATE CREEP

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The problem on equal-stressed reinforcement of metal-composite plates with fibers of constant cross section, loaded in their plane and operating under the conditions of steady-state creep, is formulated. A qualitative analysis of the corresponding system of resolving equations and boundary conditions is performed. An important case of the problem is considered, where both the reinforcing fibers and the matrix are equally stressed. A method for numerically solving this problem is developed. Particular analytical and approximate solutions are discussed, with the example of which changes in the reinforcement structure in relation to the stress level in the reinforcing fibers are clarified. It is shown that nonunique solutions to such problems can exist.

Introduction

In designing metal-composite plates, it is expedient to carry out their optimization. One of the strength criteria of rational reinforcement of thin-walled structures is the requirement that the force elements — fibers (wires) — be equally stressed along their trajectories, because the load-carrying capacity of the reinforcement is then exploited most completely.

Upon long-term operation of products, in the case of static thermal and force loading, metal-composite structures operate under the conditions of steady-state creep [1] for the overwhelming period of time; therefore, the solution to the problem of equal-stressed reinforcement (ER) of thin-walled structures operating under steady-state creep for all composite phases, with regard for the technologically important condition of constant cross sections of fibers, is topical. The present study is dedicated to investigating this problem in the case of plates loaded in their plane.

1. System of Initial Equations and Boundary Conditions

Let us consider a metal-composite plate operating under the conditions of generalized plane stress state and reinforced with \( N \) families of fibers (metal wires, possibly made of different materials) of constant cross section. We assume that, by an instant of time considered, the creep deformations have developed to such an extent that, compared with them, the initial elastic and plastic deformations can be neglected [1]. The problem of ER of such a structure in steady-state creep is described by the following equations and relations (the flow theory and the model of a reinforced layer from [2] are used):

— the equilibrium equations [1]

The parameters \( X_j \) and \( v_i \) are vector components of the reduced mass loads and creep rates of points of the plate along the \( x_i \) \((i = 1, 2)\) axes of a rectangular Cartesian system of coordinates \( (u_j) \) are displacements; the dot means differentiation with respect to the time \( t) ; \rho_0 \) and \( \rho_k \) are the volumetric densities of materials of the binder and the reinforcement of a \( k \)th family; \( X_j^0 \) and \( X_j^{(k)} \) are components of the specific mass loads acting on the binder and the reinforcement of the \( k \)th family; \( \sigma_{ij} \), \( \sigma_{ij}^0 \), and \( \xi_{ij} \) are the tensor components of average stresses, stresses in the binder, and creep strain rates, respectively; \( \sigma_k \) and \( \xi_k \) are the stress and the creep strain rate in the \( k \)th family of reinforcement, which are related by a proportionality factor \( g_k (\xi_k , \theta) \) (the relation between \( g_k \) and the quantities \( \xi_k \) and \( \theta \) is known); \( g_0 (H, \theta) \) is a given function, presenting the proportionality factor between the intensities of tangential stresses \( T \) and strain rates \( H \) in the binder: \( T = g_0 (H, \theta) H \); \( a \) is the intensity of binder interlayers; \( \omega_k \) and \( \psi_k \) are the reinforcement intensity and angle (reckoned from the \( x_1 \) direction) of the \( k \)th fiber family, with the physical restrictions