ON THE PROPAGATION OF LAMB WAVES IN A SANDWICH PLATE MADE OF COMPRESSIBLE MATERIALS WITH FINITE INITIAL STRAINS

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The propagation of flexural Lamb waves in a prestrained sandwich plate made from compressible highly elastic materials is investigated within the scope of a piecewise homogeneous body model by utilizing TLTEWISB. The mechanical relations of layer materials are described by a harmonic-type potential, and numerical results are obtained for the first and second vibration modes. According to the results, the influence of problem parameters and of the initial stretching strain along the layers on the wave propagation speed is examined. The asymptotic values of the speed are considered in the cases of short and long wavelengths, and the influence of the initial strains on these asymptotic (limit) values are also analyzed.

Introduction

The present level of modern engineering and technology requires a more detailed and accurate estimation of the dynamic carrying capacity of structural members, with taking into account their initial distinctive features. One of the features of structural members is the presence of initial (residual) stresses in them. These stresses arise in the members after their manufacture and assembly, in the Earth crust from the action of geostatic and geodynamic forces, in composites, in rocks, etc. Therefore, the studies on wave propagation in bodies with initial stresses are of great significance both for the theory and the actual practice. Up to now, a large number of investigations have been made in this field, and a considerable part of them were performed by utilizing the Three-dimensional Linearized Theory of Elastic Waves in Initially Stressed Bodies (TLTEWISB). Here, we briefly consider some of them with regard to the subject of the present paper.

The field equations of TLTEWISB were constructed in [1-3]. Moreover, in [3], detailed analyses of the results obtained and of their applications were also carried out. It follows from these references and the review [4] that wave propagation problems for prestressed multilayer cylinders and plates remain practically uninvestigated up to now. Also, the near-surface waves in initially stressed layered half-planes have been examined very poorly. The first attempts in this field were made in [5-8]. In [5], the axisymmetric wave propagation in a prestretched compound cylinder was considered, but in [6, 7] problems on the generalized Rayleigh wave dispersion in a layered half-plane were studied.
In [8], the propagation of Lamb waves in a prestrained sandwich plate made of incompressible high-elastic materials was investigated. However, the character of the investigation and discussions of the numerical results obtained in [8] do not allow one to draw any conclusion about the influence of initial strains on the propagation of Lamb waves in a sandwich plate. Since systematic investigations into the problem are absent, in the present paper, an attempt is made to bridge the gap. It is assumed that the layers of a sandwich plate, which are made of high-elastic compressible materials, are prestrained before their assembling. The mechanical properties of layer materials are described by a potential of harmonic type.

More detailed analyses of recent investigations into the dynamics of initially stressed bodies can be found in [9-17].

1. Formulation of the Problem

We consider a sandwich plate with the structure shown in Fig. 1 and assume that the thickness and materials of the face layers of the plate are the same. We differ three (natural, initial, and perturbed) states of the plate, and relate the points of the plate in the natural (initial) state to the Lagrange coordinates in a Cartesian coordinate system $Ox_1x_2x_3(y_1y_2y_3)$. In addition, the midplane of each layer of the plate in the initial state is associated with the corresponding local coordinate system $O_{n_1n_2n_3}(y_{n_1}y_{n_2}y_{n_3})$, which is obtained by a parallel transition of the coordinate system $Ox_1x_2x_3(y_1y_2y_3)$ along the $Ox_2$ ($Oy_2$) axis. The layer materials are high-elastic, and the layers are prestretched by uniformly distributed normal stresses (Fig. 1) before their assembly. As a result, initial strains in the layers, which are homogeneous, are determined by the relations

$n_i^{(r_n)0} = (\lambda_i^{(r_n)} - 1)x_{in}, \quad \lambda_i^{(r_n)} = \text{const}_{in}, \quad n_1 = n_2 = 2, \quad n_3 = 1, \quad y_{in}^{(r_n)} = \lambda_i^{(r_n)}x_{in}; \quad n, m, i = 1, 2, 3. \quad (1)

In Eq. (1) and in what follows, the conventional notation is used, and the quantities related to an $n$th layer are denoted by the superscript $(r_n)$. The quantities referring to the initial state are labeled by the superscript 0.

The elastic properties of layer materials are given by the harmonic-type potential

$$\Phi = \frac{1}{2} \lambda s_1^2 + \mu s_2, \quad (2)$$

where

$$s_1 = \sqrt{1 + 2\varepsilon_1} + \sqrt{1 + 2\varepsilon_2} + \sqrt{1 + 2\varepsilon_3} - 3,$$

$$s_2 = (\sqrt{1 + 2\varepsilon_1} - 1)^2 + (\sqrt{1 + 2\varepsilon_2} - 1)^2 + (\sqrt{1 + 2\varepsilon_3} - 1)^2. \quad (3)$$

In relation (3), $\lambda$ and $\mu$ are material constants, and $\varepsilon_i$ ($i = 1, 2, 3$) are the principal values of the Green strain tensor. When needed, expressions (2) and (3) are supplied with corresponding indices.

Taking into account the foregoing assumptions, we will investigate the propagation of Lamb waves in a sandwich plate within the scope of a piecewise homogeneous body model, by utilizing TLTEWISB under the plane strain state in the $Oy_1y_2$ plane, for which $\gamma_1^{(1)} = \gamma_2^{(2)} = 1.0$ and $\varepsilon_3^{(1)} = \varepsilon_3^{(2)} = 0.$

The field equations of TLTEWISB for the problem considered are

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