Dynamic Effective Properties of Particle-Reinforced Composites with Viscoelastic Matrix

P.J. WEI¹,²

¹LNM (State Key Laboratory of Nonlinear Mechanics), Institute of Mechanics, Chinese Academy of Sciences, Beijing, P.R. China
²School of Applied Sciences, University of Sciences and Technology Beijing, Beijing, P.R. China; E-mail: weipj@126.com

(Received 11 February 2004; accepted in revised form 11 February 2004)

Abstract. The frequency-dependent dynamic effective properties of the particle-reinforced composites with the viscoelastic matrix are studied. Several equations to predict the effective wavenumber of the coherent plane waves propagating through particle-reinforced composites are discussed and the equation given by Gubernatis, J.E., ['Effects of microstructure on speed and attenuation of elastic waves in porous materials', Wave Motion, 6, 1984, 579–589] based on the independent scattering approximation is used in this paper. The effective phase velocity, the effective attenuation and the effective elastic moduli are evaluated. Numerical calculations are carried out for two kinds of composites, namely, Lead-Epoxy and Glass-Epoxy and the numerical results show that the frequency-dependent dynamic effective properties are related to both the multiple scattering effects among the distributed particles and the viscous dissipative effects of the viscoelastic matrix. However, these effects in the composites with distributed heavy particles (lead) and light particles (glass) are of evidently different features.

Keywords: dynamic effective properties, multiple scattering, coherent plane waves, configurational average, viscoelastic matrix

1. Introduction

The determination of the effective propagation constants of waves propagating through composite materials has been a subject which attracted a considerable attention (Foldy, 1945; Lax, 1951; Varadan, 1985; Datta, 1988; Shindo, 1995; Kanaun, 2000). Foldy studied early the effective wavenumber of a scalar wave propagating through an inhomogeneous medium with distributed particles based on the multiple scattering theory. In this theory, a set of equations in hierarchy, each containing more statistical distribution information than those preceding, is involved. In order to truncate these equations to obtain an approximate solution, a self-consistent approximation is given by Foldy. Later, in order to consider better the distribution correlation between two particles the well-known “quasi-crystalline approximation” is proposed by Lax. Varadan et al. (1985) extended the multiple scattering theory of a scalar wave to the elastic waves. Further, for the elastic longitudinal and shear waves propagating through the composite medium, Gubernatis
(1984) gave the coherent plane wave equations based on the independent scattering approximation. It is evident that all the approximations mentioned above are aimed at simplifying the interaction among the particles. On the other hand, the interaction among the distributed particles can be described approximately by assuming that each particle is embedded in the effective medium. It is usually called the effective medium approach and is employed by Berryman (1980), Sabina and Willis (1988), Kanaun (2000) and others. The multiple scattering theory and the effective medium approach are based on different assumptions to simplify calculations, and thus, generally speaking, will give different results when applied to a given composite medium. Nevertheless, same qualitative conclusions are obtained, namely, the coherent plane waves are of attenuation nature although the matrix and the distributed particles are both elastic materials. If the viscoelasticity of matrix is considered, then the material dissipative effects and the multiple scattering effects are both contributed to the attenuation of waves. However, most of investigations up to now are given to elastic matrix and particles. In the present work, the material dissipation is taken into account. The different effects of the material dissipation and the multiple scattering on waves propagating through composites are our main concerns. The outline of the paper is as follows: in Section 2, the scattering problem of an elastic sphere embedded in a viscoelastic matrix is studied and the far-field scattering amplitudes are formulated. In Section 3, several equations to predict the effective wavenumbers of the coherent plane waves by using the forward scattering amplitudes of an individual particle are discussed. In Section 4, the effective propagation constants and the effective elastic moduli are studied numerically for Lead-Epoxy and Glass-Epoxy composites. The material dissipation effects and the multiple scattering effects in the composites with distributed heavy and light particles are compared. Finally, conclusions are given in Section 5.

2. The Scattered Waves by an Elastic Sphere Embedded in a Viscoelastic Matrix

Consider a spherical inclusion of radius $a$ embedded in a viscoelastic matrix. The Lamé constants and the mass densities of the inclusion are denoted by $(\lambda_1, \mu_1, \rho_1)$. Due to the viscoelasticity of matrix, the Lamé constants of matrix are complex-valued and frequency-dependent in the time harmonic case. The complex-valued and frequency-dependent Lamé constants and the mass densities of the viscoelastic matrix are denoted by $(\hat{\lambda}_2(\omega), \hat{\mu}_2(\omega), \rho_2)$. The geometry is depicted in Figure 1, where $(x, y, z)$ is the right-handed Cartesian coordinate system with the origin at the center of spherical inclusion and $(r, \theta, \phi)$ is the corresponding spherical polar coordinate. The incident plane longitudinal and shear waves (P and S waves) of circular frequency $\omega$ are assumed to propagate along $z$-axis and can be expressed as the displacement vector

$$u' = a e^{i(k_0^p z - \omega t)} + b e^{i(k_0^s z - \omega t)},$$

(1)