Kinematics analysis and statics of a 2SPS+UPR parallel manipulator

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Abstract In this paper, the kinematics and statics of a 2SPS+UPR parallel manipulator are studied systematically. First, its simulation mechanism is created, and formulae for solving the inverse/forward displacement kinematics are derived. Second, formulae for solving inverse/forward velocity and active/constrained forces are derived. Third, formulae for solving inverse/forward acceleration are derived, and a workspace is analysed. The analytic results are verified by its simulation mechanism.

Keywords Parallel manipulator · Kinematics · Active force · Constrained force · Workspace

Abbreviations

- $B, m$ the base and the moving platform
- $r_i$ the active leg and its length
- $l_i, L_i$ the sideline of $m$ and the sideline of $B$
- $P, S$ the prismatic joint and the spherical joint
- $R_1, R_2, R_3$ the revolute joints
- $U$ the universal joint with $R_1 \& R_2$
- $O, o$ the center point of $B$ and the center point of $m$
- $\{m\}$ coordinate $o$-xyz fixed on $m$
- $\{B\}$ coordinate $O$-XYZ fixed on $B$
- $b_i, B_i$ the vertices of $m$ and the vertices of $B$
- $v_{ri}$ the input velocity of active leg
- $e, E$ the distances from $b_i$ to $o$ and from $B_i$ to $O$
- $\delta_i, f_j$ the unit vectors of $r_i$ and $F_{pj}$

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\[ F, T \] the central force and torque applied on \( m \) at \( o \)
\[ F_x, F_y, F_z, T_x, T_y, T_z \] the components of \( F \) and \( T \)
\[ F_{ai} \] the active forces exerted on \( r_i \)
\[ T_d \] the active torque exerted on \( R_1 \)
\[ F_{pj}, T_p \] the constraint forces \( (j = 1, 2) \), constraint torque
\[ J, H \] the Jacobian matrix and Hessian matrix
\[ x_l, y_l, z_l, x_m, y_m, z_m, x_n, y_n, z_n \] direction cosine between \( x & X, x & Y, x & Z \)
\[ y_l, y_m, y_n \] direction cosine between \( y & X, y & Y, y & Z \)
\[ z_l, z_m, z_n \] direction cosine between \( z & X, z & Y, z & Z \)
\[ \alpha, \beta, \lambda \] rotational angles of \( m \) about \( (Z, X_1, y) \)
\[ X_o, Y_o, Z_o \] the position components of \( o \) in \{ \mathcal{B} \}
\[ V \] the forward general velocity, \( V = [v \omega]^T \)
\[ A \] the forward general acceleration, \( A = [a \varepsilon]^T \)
\[ W \] the reachable workspace
\[ \parallel, \perp \] parallel and perpendicular constraints

1 Introduction

Currently, various 4-dof (degree of freedom) parallel manipulators have attracted much attention because of their relatively large workspace, simplicity in structure, larger load-bearing capability, and easy control [1–11]. Carricato synthesized a fully isotropic 3T1R 4-dof parallel mechanism with Schoenflies motion (T is a translational motion, and R is a rotational motion) [3]. Fang and Tsai synthesized some 4-dof parallel manipulators by screw theory [4]. Li and Huang revealed some structural characteristics of the 4-dof parallel manipulators by constraint synthesis [5]. Kong and Gosselin [6], Companny [7], and Choi [8] studied various 3T1R 4-dof parallel manipulators with Schoenflies motion. Alizade [9] and Gao [10] synthesized some 4-dof parallel manipulators with parallel active limbs. Chen proposed a 2T2R 4-dof hybrid parallel manipulator [11]. Tanev analyzed forward displacement of a three-leg 4-dof parallel manipulator [12]. Some parallel manipulators possess a redundant motion due to their coupled structure constraint being high sensitive to the manufacture error [13]. From the experiments of two parallel manipulator prototypes with variable dofs at Yanshan University, we also found the redundant motion. By using a constraining leg, the redundant motion of some parallel manipulators with less than 6-dof can be removed effectively. In this aspect, Zhang and Gosselin proposed \( n \)-dof parallel mechanisms with a passive constraining leg [14]. Huang et al. studied a 3-dof Tricept with a UP passive constraining leg and a 3-dof TriVariant with a UP active constraining leg [15]. Gürsel and Bijan analyzed a 3-SPS parallel manipulator with a passive constraining spherical joint [16]. In kinematics, Huang et al. studied the first/second-order kinematic influence coefficient matrices which are proved to be Jacobian/Hessian matrices later [2]. Joshi and Tsai studied Jacobian matrix for mechanisms with less than 6-dof by screw theory [17]. Kim et al. derived a homogeneous Jacobian matrix formulation by three end-effector points [18]. Fang and Huang solved velocity/acceleration of a 3-RPS manipulator by the first/second-order kinematic influence coefficient matrices [19]. Canfield et al. analyzed the velocity of parallel manipulators by truss transformations [20]. Since each of the items in the Jacobian/Hessian matrices is the result of first/second-order partial differentiation, this imposes difficulties on kinematics analysis of 2SPS+UPR parallel manipulator. In addition, the forward pose equations of parallel manipulator generally are the implicit functions and have multi-solutions. Therefore, to solve the forward velocity/acceleration of this parallel manipulator by means of the first/second-order kinematic influence coefficient matrix approach is quite complex.