Solving the Dynamic User Optimal Assignment Problem Considering Queue Spillback

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Abstract This paper studies the dynamic user optimal (DUO) traffic assignment problem considering simultaneous route and departure time choice. The DUO problem is formulated as a discrete variational inequality (DVI), with an embedded LWR-consistent mesoscopic dynamic network loading (DNL) model to encapsulate traffic dynamics. The presented DNL model is capable of capturing realistic traffic phenomena such as queue spillback. Various VI solution algorithms, particularly those based on feasible directions and a line search, are applied to solve the formulated DUO problem. Two examples are constructed to check equilibrium solutions obtained from numerical algorithms, to compare the performance of the algorithms, and to study the impacts of traffic interacts across multiple links on equilibrium solutions.

Keywords Dynamic user optimal traffic assignment • Dynamic network loading • Variational inequality • Feasible direction algorithms

1 Introduction

Predicting the temporal and spatial traffic evolution over road networks has attracted numerous research efforts since the 1970s, when transportation researchers began to recognize the limitations of static network equilibrium models (Wardrop 1952, Beckmann et al. 1956) in describing a system that essentially varies over time. These limitations include, among others, the inability of modeling departure time decisions and the formation and dissipation of queues when travel demands temporarily exceed road capacities at different locations, two important elements to consider in traffic congestion management.

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The earliest work to bring the time dimension into equilibrium analysis is perhaps Vickrey’s seminal paper on the morning commute problem (Vickrey 1969), in which traffic congestion takes the form of queuing behind a bottleneck caused by temporal demand surge. Vickrey’s bottleneck model stimulated numerous studies on the morning commute problem. Most of these studies focused on finding analytical equilibrium solutions and/or exploring economic and policy insights (e.g., Mahmassani and Herman 1984, Newell 1987, Arnott et al. 1990, Kuwahara 1990, Yang and Huang 1997). However, these models do not intend to solve, thereby are not typically applicable to the general dynamic user optimal (DUO) assignment problem, where multiple origin–destination (O–D) pairs, route/departure time choices and realistic representation of traffic dynamics are considered simultaneously.

It was not until the early 1990s that variational inequality (VI) is employed to provide a general DUO formulation, in the context of dynamic traffic assignment (DTA). Friesz et al. (1993) and Smith (1993) are the first to establish the equivalence between the dynamic user optimal condition and the solution to a corresponding variational inequality VI(c, Ω), where c represents a mapping from path flow to path cost and Ω depicts a feasible set of path flows. Nevertheless, the original VI formulation of Friesz et al. (1993) is notoriously hard to solve. Above all, the analytical properties of the nested path cost function c, such as differentiability and monotonicity, are difficult to assert (Friesz et al. 2001). This drawback hinders the design of effective solution procedures. Numerous formulations had since been proposed in the hope of casting the flow-cost mapping in a more tractable way. Most existing work are either based on VI (e.g., Wei et al. 1995, Ran et al. 1996, Chen and Hsueh 1998, Lo and Szeto 2002, Szeto and Lo 2004), or highly related to it (e.g., Huang and Lam 2002, Wei et al. 2002). In this body of work, various network traffic flow models were used, which include the delay function model (e.g., Friesz et al. 1993, Ran et al. 1996), the point-queue model (e.g., Smith 1993, Huang and Lam 2002), and the kinematic wave model (e.g., Lo and Szeto 2002, Szeto and Lo 2004). In spite of these efforts, however, the evaluation of c still relies frequently on a numerical procedure known as dynamic network loading (DNL). The complexity of DNL varies substantially according to the underlying representation of traffic flow, which conceivably has a dominant impact on the DUE solutions. Compared to DNL, designing solution procedures for dynamic traffic assignment is less well studied: heuristic algorithms were suggested in most cases, such as the method of successive averages (Tong and Wong 2000). Lo and Szeto (2002) solved the DUO problem using an alternating direction algorithm. By introducing Lagrangian multipliers, they simplified the projection used to generate the feasible direction. Recently, Friesz and Mookherjee (2006) proposed a projection algorithm which converges to a DUO solution when the nested path cost function is monotonic.1

In this paper, the DUO problem is formulated as a discrete VI while traffic dynamics is encapsulated in a mesoscopic DNL model. The formulation considers individuals’ route and departure time choices simultaneously. Our DNL model describes traffic evolution on links through macroscopic traffic models based on the kinematic wave theory, and considers link interactions by incorporating proper node models. It is thus capable of capturing realistic traffic phenomena such as the propagation of shockwaves and queue spillback.

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1 It is noted (Friesz and Mookherjee 2006), however, that this condition is unlikely to be met in general.