Enhanced nonlinear 3D Euler–Bernoulli beam with flying support

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1 Introduction

Beams are one of the most important structural elements in engineering fields. To provide the structure with less weight makes designer choose more flexible beams. As the flexibility of the beam increases, its dynamic modeling becomes more complicated in return.

Karray et al. [1] have treated the two flexible links of space-based flexible manipulator as Euler–Bernoulli beams, free to deform transversely in the orbital plane. Hiller [2] has considered only three elastic degrees of freedom for each link as an Euler–Bernoulli beam. Shi et al. [3] have modeled a planar flexible link by an Euler–Bernoulli beam. Chen [4] has presented a linearized dynamic model for multilink planar flexible manipulators which can include an arbitrary number of flexible links. Flexible links are treated as Euler–Bernoulli beams and the rotary inertia and shear deformation are thus neglected. Bruno and Luigi [5] have modeled planar n-link flexible manipulators in accordance with Euler–Bernoulli beam. Jen et al. [6] have obtained dynamic model of a one-link flexible robot, using planar Euler–Bernoulli beam. Zohoor and Khorsandijou [7] have dynamically modeled a mobile robot with long and short spatially flexible links experiencing considerable and negligible elastic orientation in their cross-sectional frames. They have exposed the dynamic model of a flying manipulator with two highly flexible links [7]. The nonlinear 3D Euler–Bernoulli beam theory has been formulated for large elastic orientation of cross-section [7], and has

Abstract Using Hamilton’s principle the coupled nonlinear partial differential motion equations of a flying 3D Euler–Bernoulli beam are derived. Stress is treated three dimensionally regardless of in-plane and out-of-plane warplings of cross-section. Tension, compression, twisting, and spatial deflections are nonlinearly coupled to each other. The flying support of the beam has three translational and three rotational degrees of freedom. The beam is made of a linearly elastic isotropic material and is dynamically modeled much more accurately than a nonlinear 3D Euler–Bernoulli beam. The accuracy is caused by two new elastic terms that are lost in the conventional nonlinear 3D Euler–Bernoulli beam theory by differentiation from the approximated strain field regarding negligible elastic orientation of cross-sectional frame. In this paper, the exact strain field concerning considerable elastic orientation of cross-sectional frame is used as a source in differentiations although the orientation of cross-section is negligible.

Keywords 3D Euler–Bernoulli beam theory

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been indirectly improved for negligible elastic orientation of the cross-section [7].

In this paper, partial differential equations of motion of a 3D Euler–Bernoulli beam with a six DOF flying support is obtained. The equations are more accurate than that of a conventional nonlinear 3D Euler–Bernoulli beam, because variation of strains and variation of elastic potential energy are derived from the exact strain field concerning considerable elastic orientation of the cross-sectional frame. The elastic orientation of the cross-sectional frame is negligible and the equations will be finally expressed by the approximated strain field. In the motion equations, two additional elastic terms have appeared that would perish in the conventional nonlinear 3D Euler Bernoulli beam theory. In this theory, variation of strains and variation of elastic potential energy are derived from approximated strain field regarding negligible elastic orientation of beam cross-sectional frame [8].

2 Flying support of the beam

The flying support of the beam has six degrees of freedom. In Fig. 1, the frame of the flying support of the beam is denoted by $F_B$. The inertial reference frame is shown by $F_I$, the direction of whose third axis is in the negative direction of the gravity. Position, variation of position, velocity, and acceleration of the flying support are projected onto $F_I$ as expressions (1). $F_B$ and $F_I$ are orthogonal and right-handed coordinate reference frames and their axes are marked by 1, 2, and 3 in the figures to indicate, the first, second, and third axes, respectively.

$$ b = [x \ y \ z]^T, \quad \delta b = [\delta x \ \delta y \ \delta z]^T, $$
$$ V^B = [\dot{x} \ \dot{y} \ \dot{z}]^T, \quad a^B = [\ddot{x} \ \ddot{y} \ \ddot{z}]^T. $$

Orientation of the flying support relative to inertial reference frame is described by three Euler angles as expression (2).

$$ R = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix} \begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix} \begin{bmatrix}
\cos \psi & -\sin \psi \\
\sin \psi & \cos \psi \\
0 & 0 & 1
\end{bmatrix}. $$

Virtual rotations and angular velocity of the flying support that are respectively imperfect differentials and nonintegrable time-derivatives are given in

![Fig. 1 Six dependent spatial elastic coordinates](image)