Isolated large amplitude periodic motions of towed rigid wheels

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Abstract This study investigates a low degree-of-freedom (DoF) mechanical model of shimmying wheels. The model is studied using bifurcation theory and numerical continuation. Self-excited vibrations, that is, stable and unstable periodic motions of the wheel, are detected with the help of Hopf bifurcation calculations. These oscillations are then followed over a large parameter range for different damping values by means of the software package AUTO97. For certain parameter regions, the branches representing large-amplitude stable and unstable periodic motions become isolated following an isola birth. These regions are extremely dangerous from an engineering point of view if they are not identified and avoided at the design stage.

Keywords Wheel · Shimmy · Bifurcation · Continuation · Stability

1 Introduction

Shimmy is a common name for the lateral vibration of a towed wheel. This has been a well-known phenomenon in vehicle systems dynamics for several decades: the name shimmy comes from a dance that was popular in the 1930s. One of the early scientific studies of shimmy dates back to this time (see [1]). In many cases, the appearance of shimmy is a serious problem, for example, in case of nose gears of airplanes or front wheels of motorcycles. There are many mechanical models (see, for example, [2–7]) that describe the shimmy of rolling wheels. The two most important considerations of any model involve whether the wheel is rigid or elastic and whether the suspension system is rigid or elastic. The simplest combination of a rigid wheel and a rigid suspension system does not, by definition, support lateral vibrations. Lateral vibration can occur in models with an elastic wheel and a rigid suspension (see [8]) and in models where both the wheel and the suspension system are elastic. But in this paper we wish to make analytic progress in order to obtain a clear understanding of the dynamics and so we consider a low degree of freedom (DoF) mechanical model of a rigid wheel, which has a viscously damped elastic suspension. This model was studied without damping in [9], where subcritical Hopf bifurcations and chaotic and transient chaotic oscillations were found.

This paper is structured as follows. First, in Sect. 2, the mechanical model is introduced, together with the
equations of motion. In Sects. 3 and 4, the Hopf bifurcation calculation is presented in the presence of viscous damping at the suspension. The effect of damping on the stability of stationary rolling is analyzed. In Sect. 5, the periodic solutions of the system are followed using AUTO97 [10], the stability charts and the bifurcation diagrams are plotted and also compared to available analytical results.

The bifurcation diagrams show an isola birth where isolated large amplitude stable and unstable periodic motions coexist with the stable stationary rolling solution. These motions are difficult to detect either by numerical simulation or by conventional stability and bifurcation analysis. The presence of unstable periodic motions indicates a dangerous system configuration.

2 Mechanical model

The mechanical model under consideration is shown in Fig. 1. The plane of the rigid wheel is always vertical to the ground, contacting at a single point P. The radius of the wheel is $R$, the mass of the wheel is $m_w$ and its mass moment of inertia with respect to the $z$ axis is $J_{wz}$. The mass moment of inertia with respect to the $y$ axis of its rotation is $J_{wy}$, where the subscript $w$ refers to the wheel. The caster length is $l$, and the distance between the center of gravity $C$ of the caster and the king pin at $A$ is $l_c$. The mass of the caster is $m_c$ and the mass moment of inertia with respect to the $z$ axis at $C$ is $J_{cz}$, with the subscript $c$ referring to the caster. The system is towed in the horizontal plane with constant velocity $v$. The king pin is supported by lateral springs of overall stiffness $k$ and viscous damping coefficient $b$.

Without rolling constraints, the system has 3 degrees of freedom, so one can choose the caster angle $\psi$, the king pin lateral position $q$, and the wheel rotation angle $\varphi$ as general coordinates. The constraint of rolling (without sliding) means that the contact point $P$ has zero velocity. This rolling condition leads to two scalar kinematical constraint equations in the form of coupled first order nonlinear ordinary differential equations (ODEs) with respect to the general coordinates.

The equations of motion of this rheonomic and nonholonomic system can be derived with the help of the Routh–Voss equations or the Appell–Gibbs equations [11]. In the case of zero damping ($b = 0$), they are given in [12]. Here, we present the equations for nonzero viscous damping: