Differential properties and optimality conditions for generalized weak vector variational inequalities

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Abstract In this paper, we study a generalized weak vector variational inequality, which is a generalization of a weak vector variational inequality and a Minty weak vector variational inequality. By virtue of a contingent derivative and a \( \Phi \)-contingent cone, we investigate differential properties of a class of set-valued maps and obtain an explicit expression of its contingent derivative. We also establish some necessary optimality conditions for solutions of the generalized weak vector variational inequality, which generalize the corresponding results in the literature. Furthermore, we establish some unified necessary and sufficient optimality conditions for local optimal solutions of the generalized weak vector variational inequality. Simultaneously, we also show that there is no gap between the necessary and sufficient conditions under an appropriate condition.

Keywords Contingent derivative · Gap function · Generalized weak vector variational inequality · Optimality conditions

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1 Introduction

The vector variational inequality (VVI, in short) and weak vector variational inequality (WVVI, in short) were firstly introduced by Giannessi [8] in a finite-dimensional spaces. These problems have been of great interest in the academic and professional communities in the last few decades. Various kinds of variational inequalities have been discussed and a lot of important results have been established. Especially, a great deal of researches on the existence of solutions (see [4, 6, 19]) and the stability of the solution set map (see [13, 14]) have been obtained.

To the best of our knowledge, the concept of gap function is very useful to study (VVIs). Until now, there were two kinds of gap function introduced to (VVIs). Yang and Yao [19] introduced a kind of gap function for (VVIs) as a real-valued function. In [5], Chen et al. defined another kind of gap function for (VVIs) as a set-valued map from the vector optimization point of view. Moreover, Li et al. [15] discussed the differential and sensitivity properties of the set-valued gap function for (VVIs) and (WVVIs) and obtained an explicit expression of the contingent derivative for a class of set-valued maps, and some necessary optimality conditions under some suitable coerciveness condition, respectively. In [16], Li and Zhai introduced a asymptotic second-order $\Phi$-contingent cone, and discussed the second-order asymptotic differential properties and some necessary optimality conditions for (WVVIs).

Recently, the Minty vector variational inequality (MVVI, in short) and Minty weak vector variational inequality (MWVVI, in short) have also received extensive attentions, and many important results have been established. Giannessi [9] investigated some relationships between a solution of a (MVVI) and an efficient solution or a weakly efficient solution of a vector optimization problem under convexity and monotonicity conditions. Subsequently, Yang et al. [18] established some relations between a (MVVI) and a vector optimization problem under pseudoconvexity or pseudomonotonicity conditions, respectively. In [17], Meng and Li introduced a $\Phi$-contingent cone and obtained an explicit expression of the contingent derivative for a class of set-valued maps without any coerciveness condition. They also defined a kind of gap function for (MVVIs) and (MWVVIs) as set-valued maps, discussed the differential and sensitivity properties of gap functions and got some necessary optimality conditions for (MVVIs) and (MWVVIs), respectively.

It is well known that the (VVI) is closely related to vector optimization problems [8, 9, 12], vector complementarity problems [7], vector equilibria problems [20] and so on. Moreover, there are many real world applications, such as economic or engineering problems, can be modeled by means of the (VVI) and some of its variants. Recently, the (MVVI) has been regarded as a dual form of the (VVI) based on its close relationship to the classical vector variational inequality. It has been shown, in [9, 10, 18], that the (MVVI) has a lot of important applications to standard optimality topics by using some generalized convexity and monotonicity assumptions. Motivated by the work reported in [9, 15–17, 19], we investigate a generalized weak vector variational inequality (GWWVI, in short) in this paper, which is more extensive