Interpreting and testing the scaling property in models where inefficiency depends on firm characteristics

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Abstract Let \( u \geq 0 \) be technical inefficiency, let \( z \) be a set of variables that affect \( u \), and let \( \delta \) be the parameters of this relationship. The model satisfies the scaling property if \( u(z, \delta) \) can be written as a scaling function \( h(z, \delta) \) times a random variable \( u^* \) that does not depend on \( z \). This property implies that changes in \( z \) affect the scale but not the shape of \( u(z, \delta) \). This paper reviews the existing literature and identifies models that do and do not have the scaling property. It also discusses practical advantages of the scaling property. The paper shows how to test the hypothesis of scaling, and other interesting hypotheses, in the context of the model of Wang, *Journal of Productivity Analysis*, 2002. Finally, two empirical examples are given.

Keywords Stochastic frontier model · Scaling property · Technical inefficiency

JEL Classification C12 · C31 · C52

Introduction

In this paper, we are interested in a stochastic frontier model in which observable characteristics of the firms affect their levels of technical inefficiency. To be more precise, let \( u \geq 0 \) be the one-sided error reflecting technical inefficiency, and let \( z \) be a set of variables that affect \( u \). Then we can write \( u \) as \( u(z, \delta) \) to reflect its dependence on \( z \) and some parameters \( \delta \). Various models in the existing literature specify the distribution of \( u(z, \delta) \). We will be interested in models that satisfy the scaling property, which says that \( u(z, \delta) \) can be written as a scaling function \( h(z, \delta) \) times a random variable \( u^* \) that does not depend on \( z \). This property implies that changes in \( z \) affect the scale but not the shape of \( u(z, \delta) \).

We discuss scaling in the context of the stochastic frontier model but it is also of relevance in the semiparametric (DEA) context. There is a large literature attempting to relate DEA efficiency scores to “environmental variables” \( z \). Simar and Wilson (2003) give a survey of this literature, and they introduce a data generating process in which the (population) output efficiency is random and depends in a parametric way on some variables \( z \). Their assumption about the way efficiency scores depend on \( z \) corresponds to the KGMHLBC model discussed...
below. They could instead have assumed a scaling model. Some (but not all) of the models in the literature have the scaling property. However, there is really no previous systematic treatment of scaling as a unifying principle. (The article that comes closest is Simar, Lovell and van den Eeckaut (1994).) In this paper, we provide a comprehensive treatment of the scaling property, and a review of the relevant literature. We identify models in the literature that do and do not have this property, and we propose a specific model that may be empirically useful. We discuss the practical advantages of models with the scaling property. We also show how to test the scaling hypothesis, and other interesting hypotheses, in the context of the model of Wang (2002).

The paper also makes the following important observation, which is known in the econometric literature but appears to have been missed in the production frontier literature. Maximum likelihood estimates of models that assume independence of technical efficiency over time (such as our model, and most models that deal with determinants of inefficiency) remain consistent even if the independence assumption is false. However, the estimated variances of the parameters using the usual formulas that assume independence are incorrect. We show in this paper how they can be corrected in a simple way. This affects tests for scaling, as well as any other inference based on the estimated coefficients.

The plan of the paper is as follows. In Section “The scaling property”, we present our basic framework. In Section “Review of literature” we review some of the existing literature and we identify models that do and do not have the scaling property. In Section “Advantages of scaling property”, we discuss the practical advantages of models with this property. We also discuss corrections for non-independence when the MLE is based on an incorrect assumption of independence. In Section “Testing scaling and other interesting hypotheses”, we discuss tests of the hypothesis that the scaling property holds, and of other interesting hypotheses that allow us to distinguish between various competing models. In Section “Empirical examples”, we give two empirical examples involving Spanish banks and Indian farms. The last section is concluding remarks.

The scaling property

Our basic setup and notation follows Wang and Schmidt (2002). We suppose that we have panel data, in which firms are indexed by \( i = 1, \ldots, N \) and time is indexed by \( t = 1, \ldots, T \). Let \( y_{it} \) be log output; let \( x_{it} \) be a vector of variables that affect the position of the frontier; and let \( z_{it} \) be a vector of variables that affect the magnitude of technical inefficiency. Generally the \( x_{it} \) are inputs and the \( z_{it} \) are either functions of inputs or measures of the environment in which the firm operates. The \( x_{it} \) and \( z_{it} \) can overlap. Because the \( z_{it} \) (like the \( x_{it} \)) are treated as “fixed,” they cannot be functions of \( y_{it} \).

Let \( y_{it}^* \geq y_{it} \) be the unobserved frontier. The linear stochastic frontier model asserts that, conditional on \( x_{it} \) and \( z_{it} \), \( y_{it}^* \) is distributed as \( N(x_{it}'\beta, \sigma^2_v) \). Then we can write the frontier as:

\[
y_{it}^* = x_{it}'\beta + v_{it} \tag{1}
\]

where \( v_{it} \) is distributed as \( N(0, \sigma^2_v) \) and is independent of \( x_{it} \) and \( z_{it} \). Finally, the model asserts that, conditional on \( x_{it} \) and \( z_{it} \), \( y_{it}^* \) equals \( y_{it}^* \) minus a one-sided error whose distribution depends only on \( z_{it} \). Therefore we can write the model as:

\[
y_{it} = x_{it}'\beta + v_{it} - u_{it}(z_{it}, \delta), \quad u_{it}(z_{it}, \delta) \geq 0. \tag{2}
\]

Here \( u_{it} \) and \( v_{it} \) are independent of each other and of \( x_{it} \), and in addition \( v_{it} \) is independent of \( z_{it} \).

We will say that the model has the scaling property if

\[
u_{it}(z_{it}, \delta) = h(z_{it}, \delta) \cdot u_{it}^*, \tag{3}
\]

where \( h(z_{it}, \delta) \geq 0 \), and where \( u_{it}^* \geq 0 \) has a distribution that does not depend on \( z_{it} \). We will call \( h(z_{it}, \delta) \) the scaling function and \( u_{it}^* \) the basic random variable, while the distribution of \( u_{it}^* \) will be called the basic distribution.

The essential feature of the scaling property is the fact that changes in \( z_{it} \) change the scale but not the shape of the distribution of \( u_{it} \). This is so because the shape is determined by the basic distribution, which does not depend on \( z_{it} \), whereas the scaling function \( h(z_{it}, \delta) \) determines the scale. More precisely, suppose that \( u^* \) has density \( f(u^*) \), and that \( u = h \cdot u^* \) where \( h \) is treated as a constant. Then the density of \( u \) equals \((1/h)f(u/h)\). The sense in which