Reduced Form Mortgage Pricing as an Alternative to Option-Pricing Models

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Abstract This paper extends the traditional hazard technique of estimating prepayment and default by allowing their baselines to be stochastic processes, rather than known paths of time, as is typically assumed. By working in the reduced form, this method offers an alternative to the empirical valuation of mortgages more easily implemented than the standard structural form approach of options pricing.

Keywords Reduced form pricing · Mortgage valuation · Prepayment · Default

Introduction

Hazard models would not, on the face of it, seem to be a new approach to analyzing mortgages at all; indeed they are already the standard technique employed to empirically investigate prepayment and default behavior. Traditionally, with proportional hazard models, one specifies a number of explanatory variables describing loan or borrower characteristics, and then studies how these covariates shift the baseline, i.e., the common, underlying pattern of loan termination over time. Therefore, the instantaneous likelihood of, say, default, $\lambda(t)$, would be expressed as $\lambda(t) = e^{x(t)}$, where $x(t)$ is the vector of possibly time-varying covariates and $\lambda_0(t)$ is the baseline, common to the various mortgages.

1Thus the instantaneous likelihood of, say, default, $\lambda(t)$, would be expressed as $\lambda(t) = e^{x(t)}\lambda_0(t)$, where $x(t)$ is the vector of possibly time-varying covariates and $\lambda_0(t)$ is the baseline, common to the various mortgages.

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One barrier to valuation is that, before pricing can be done, each source of randomness in the model has to be converted from real form to its “risk-neutral” form as seen by the market; however, this is a relatively well-understood issue. This observation, though, does raise the question of just what are the underlying sources of uncertainty in such a hazard model. A little thought shows that the uncertainty can be of but two kinds: firstly, the idiosyncratic risk that a house will, say, default, given the hazard, and secondly, any randomness arising because covariates of the default hazard contain other stochastic elements external to the hazard. Now, given the deep market for mortgages, the idiosyncratic default risk for a single loan should be in large part diversifiable, which would leave only the uncertainty within the covariates to require much of a risk premium. In practice, though, the only process available to drive these observable covariates is likely to be the term structure, suggesting that the answer as to why current hazard models are not used for valuation purposes is simply that they provide too little systematic randomness in order to explain the required spreads observed between mortgages and, say, the long rate on Treasury bonds.

Turning to the contingent-claims literature in finance, an alternative to the usual option-based pricing of corporate default has arisen in recent years, often termed the reduced-form approach. This method is also hazard-based, but supposes that not only is there individual risk that a loan will terminate given the hazard rate, but that the hazard rate itself should be random. The resulting models are sometimes then referred to as being doubly stochastic.

Now, strictly speaking, a proportional hazard model with some of its covariates random would then be doubly stochastic, but as we have observed above, usual formulations using only the term structure as a source of uncertainty are likely to prove empirically inadequate. A device then, natural within the reduced-form framework, is to suppose that, in addition to any randomly varying covariates, the baseline hazard of the proportional hazard process should itself be stochastic. Note that, in contrast to the option-based, structural approach usually employed to explain default, one need not be overly concerned with exactly why it is that such variation in mortgage termination occurs; it suffices that it does. This is obviously a particularly appropriate viewpoint when modelling prepayment, since it has long been observed that much prepayment is “sub-optimal”, in the sense that whatever the reasons for it occurring, they are apparently not to minimize the market value of the loan.

Since, compared to the options-pricing method, the reduced-form approach abandons precise endogenous explanations as to why termination occurs, for it to be useful, it has to lend itself to estimation, as indeed it will be seen that it does. However, to better appreciate the distinction between the usual deterministic hazard rates and the alternative stochastic ones being introduced, we will first pursue the analogy between a stochastic hazard structure and a stochastic term structure, trusting that the reader will be familiar with the latter.

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2 See, for instance, Duffie (2001).
3 This is formally treated by Jarrow, Lando, and Yu (2003).
4 Another way to state the above is that, without the risk in the baseline to be introduced below, the remaining sources of risk would require implausibly high premia in order to explain observed spreads.