ELEMENARY PARTICLE PHYSICS AND FIELD THEORY

THE EFFECTIVE ACTION FOR SUPERFIELD LAGRANGIAN QUANTIZATION IN REDUCIBLE HYPERGAUGES

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The rules of local superfield Lagrangian quantization in reducible non-Abelian hypergauge functions are formulated for an arbitrary gauge theory. The generating functionals of standard and vertex Green’s functions which depend on the Grassmann variable \( \eta \) via super(anti)fields and sources are constructed. The difference between the local quantum and the gauge fixing action determines an almost Hamiltonian system such that translations with respect to \( \eta \) along the solutions of this system define the superfield BRST transformations. The Ward identities are derived and the gauge independence of the S-matrix is proved.

INTRODUCTION

The BRST symmetry principle underlies both the canonical [1] and the covariant quantization scheme [2] for general gauge theories and their superfield generalizations [3–5]. The superfield quantization [3], which is applicable in the canonical formalism and in its implication – Lagrangian formalism, makes use of the nontrivial relation of the odd Grassmann \( \eta \) and even \( t \) projections of supertime, \( \Gamma = (t, \eta) \), as distinct from the Lagrangian quantization [4, 5].

Note that algorithmic methods have been found [6] for constructing generalized Poisson sigma models in the framework of the superfield formalism [7]. The quantization described in [3] has been generalized for two and more supersymmetries which are associated with Grassmann variables \( \eta_1, \ldots, \eta_N \) [8].

The aim of the work under consideration is to construct a local version of the superfield Lagrangian quantization (SLQ). In the SLQ, we realize the explicit superfield representation of the structural functions of a gauge algebra (GA), not indicated in [4, 5], in the framework of an \( \eta \)-local superfield model (SM) which includes the initial standard gauge model.

A correlation with classical mechanics reconstructs the dynamics and gauge invariance for the original model in terms of \( \eta \)-local differential equations (DE’s). The properties of the local generating functionals of Green’s functions (GFGF’s) are derived from a Hamiltonian system (HS), which is constructed w.r.t. the \( \eta \)-local quantum and the gauge fixing action.

In the SLQ, we first define the effective action for a wider class of non-Abelian reducible hypergauges (the case of irreducible hypergauge functions is considered in [9]).

In this paper, we describe the Lagrangian and Hamiltonian formulations of the SM, specify quantization rules, and determine, based on the component formulation, the relations of the proposed quantization scheme to the superfield quantization [4, 5] and multilevel formalism [9].

We make use of some of conventions from [4, 5] and the condensed notation from [10]. The rank of an even supermatrix is characterized by a pair of numbers \( (k_+, k_-) \), where \( k_+ \) and \( k_- \) are the respective ranks of the Bose–Bose and Fermi–Fermi blocks of the supermatrix with respect to the basic Grassmann parity \( \varepsilon \). A similar pair of numbers denotes the dimension of a supermanifold, which is equal to \(( \mathrm{dim}_{-1}, \mathrm{dim}_{-2} )\). On the set of these pairs, operations of component-wise composition and comparison \( ((k_+, k_-)>(l_+, l_-) \Leftrightarrow (k_+\geq k_-, l_+>l_-) \) or \( (k_\geq k_-, l_+>l_-) \), \( (k_+, k_-)=(l_+, l_-) \Leftrightarrow (k_+=l_+, k_-=l_-) \) are defined.
THE LAGRANGIAN AND HAMILTONIAN FORMULATIONS OF A SUPERFIELD MODEL

The basic objects in the Lagrangian and Hamiltonian formulations of an SM are the Grassmann-valued $C^\infty(\Pi TM_{cl})$ and $C^\infty(\Pi^*M_{cl})$ functions of the respective Lagrangian and Hamiltonian actions$^1$:

$$S_L : \Pi TM_{cl} \times \{\eta\} \to \Lambda_1(\eta, R), \quad S_H : \Pi^*M_{cl} \times \{\eta\} \to \Lambda_1(\eta, R), \quad \epsilon(S_L) = \epsilon(S_H) = \theta$$

(1)

and the functionals $Z[A]$ and $Z_0[\Gamma]$, nonequivalent to these functions, whose densities are defined accurate to the corresponding functions $f(A(\eta), \partial_\eta A(\eta), \eta)$ and $f(\Gamma_i(\eta), \eta) \in \text{Ker}(\partial_\eta)$, (1) $d\eta = \partial_\eta$, $\partial_\eta \equiv \frac{d}{d\eta}$:

$$(Z[A], Z_H[\Gamma_i]) = \left(\partial_\eta S_L(\eta), \partial_\eta \left(\frac{1}{2} \Gamma^p_\eta \omega_{pq}^\eta \partial_\eta \Gamma^q_\eta - S_H(\eta)\right)(\eta)\right), \quad \epsilon(Z) = \epsilon(Z_H) = (1, 0, 1).$$

(2)

The values of the Grassmann parities $\epsilon = (\epsilon_p, \epsilon_\eta, \epsilon)$, $(\epsilon = \epsilon_p + \epsilon_\eta)$, where $\epsilon_\eta$ and $\epsilon_p$ are auxiliary $Z_2$-gradings w.r.t. the coordinates $z^M$ and $\eta$ of the superspace$^2$ $M = M \times \tilde{P}$, are defined for $A'(\eta)$ by the rule $\epsilon(A') = ((\epsilon_p)_l, (\epsilon_\eta)_l, \epsilon_l)$. The supermatrix $\omega_{pq}^\eta(\eta)$ is inverse of the supermatrix $\omega_{pq}^\eta(\eta) = [\Gamma^p_\eta(\eta), \Gamma^q_\eta(\eta)]$, which is defined in terms of the local superantibracket $(\cdot, \cdot, \cdot)_\eta = \frac{\partial_\eta \cdot \omega_{pq}^\eta(\eta)}{\partial_\eta \Gamma^q_\eta(\eta)}$, where $\frac{\partial_\eta \cdot}{\partial_\eta \Gamma^q_\eta(\eta)}$ is the right (left) superfield variational derivative w.r.t. the superfield $\Gamma^p_\eta(\eta)$ for a fixed $\eta$.

Assuming the existence of a critical superfield configuration for the functionals $Z[A]$ and $Z_0[\Gamma]$, we may code the SM dynamics, respectively, by superfield Euler–Lagrangian equations, which, in view of the identities $\partial_\eta^2 A'(\eta) = 0$, are equivalent to a Lagrangian system (LS), and by a Hamiltonian system (HS)

$$\begin{align*}
\partial_\eta^2 A'(\eta) - \frac{\partial_\eta^2 S_L(\eta)}{\partial \eta A'(\eta)} \partial_\eta A'(\eta)(S_L^\eta(\eta)) = 0,

\Theta_i(\eta) = \frac{\partial_\eta S_L(\eta)}{\partial A'(\eta)} - (-1)^{\epsilon_i} \left(\partial_\eta \frac{\partial_\eta S_L(\eta)}{\partial \eta A'(\eta)} + \left(\partial_\eta U_+(\eta)\right) \frac{\partial_\eta S_L(\eta)}{\partial \eta A'(\eta)}\right) = 0;

\partial_\eta^2 \Gamma^p_\eta(\eta) = \left(\Gamma^p_\eta(\eta), S_H(\eta)\right)_{\eta}.
\end{align*}$$

(3)

(4)

$^1$ Here, $\Pi TM = \{(A', \partial_\eta A')(\eta) | A'(\eta) = (A' + \lambda' \eta) \in M_i, i = 1, \ldots, n = n_+ + n_-\}$ and $\Pi^*M = \{(\Gamma^p_\eta(\eta) = (A', A^*')(\eta) | A'(\eta) = A' - \eta J, i = cl \}$ are the odd tangent and cotangent bundles over the configuration space of classical superfields $A'(\eta)$ with $n_+$ bosonic and $n_-$ fermionic degrees of freedom with respect to the $\epsilon$-parity for fixed continuous components of the condensed index $i$, and the quantities $A'$, $\lambda'$, $A^*$, and $J_i$ for $\epsilon_p(A'(\eta)) = 0$ are, respectively, the classical fields, Lagrangian multipliers, antifields, and sources to the fields $A'$ of the Batalin–Vilkovisky (BV) quantization method.

$^2$ The superfields $\Gamma^p_i(\eta)$ are defined on $M$, which can be realized as the quotient of a global symmetry supergroup $J$ for the functionals $Z[A], Z_0[\Gamma], J = \tilde{J} \times P, P = \exp\{\mu p_\eta\}$, where $\mu$ and $p_\eta$ are the nilpotent parameter and the generator of $\eta$-shifts, respectively ($\mu^2 = p_\eta^2 = 0$), with a spacetime supersymmetry group chosen for $\tilde{J}$.