RADIATIVE LOSSES OF AN OPTICAL WAVEGUIDE WITH A ROUGH SURFACE

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The main and well studied components of integrated optical circuits are optical waveguides of different types [1–3]. The complexity of constructing mathematical models of integrated optical devices is largely caused by the roughness of the optical waveguide film surface being on the average even. Irregularities of the optical waveguide surface result in the interaction of waveguide and radiation modes in the waveguide and finally determine the level of the nondissipative radiative absorption of waveguide modes. Two approaches to the determination of the attenuation coefficient of waveguide modes $\alpha$ caused by the film surface roughness are most widespread in the literature. They are the method of effective current sources [1, 4] and the method of cross sections [2]. The optical waveguide model based on the above-indicated approaches neglects the backward effect of the radiation modes on the waveguide field; therefore, it is self-inconsistent.

The present study is aimed at application of the method of coupled waves for the construction of a self-consistent model of radiative losses in planar optical waveguides with two-dimensional irregularities of the film surface.

Let us consider a single-mode planar optically-linear waveguide without losses and with a stepwise profile of the refractive index located in the $YOZ$ plane with the TM waveguide mode $E_s(r)$ propagating in the $z$-axis direction. The film thickness is $d$, and the normal to the optical waveguide is denoted by $n$. The amplitude, polarization, normalized profile, and wave vector of the waveguide mode are designated by $A$, $e$, $E_s(x)$, and $k_s$, and the refractive indices of the film, substrate, and cladding of the optical waveguide are designated by $n_f$, $n_s$, and $n_c$, respectively. Let us assume that many random microscopic irregularities – local spatial fluctuations of the film surface – are concentrated on the optical waveguide layer surface. Based on the method of effective refractive index [3], these irregularities can be characterized by two-dimensional random refractive index fluctuations of the flat surface optical waveguide layer with thickness $d$:}

$$n(r) = n_{nf} + \Delta n \cdot f(r),$$

where $r$ is the radius vector lying in the $YOZ$ plane of the film, $f(r)$ is a uniform isotropic stochastic function describing the two-dimensional roughness field of the film.

From the physical viewpoint, the process of waveguide mode attenuation in the optical waveguide with rough nonabsorbing film is a transformation of the waveguide mode into the continuum of radiation modes. In this case, the intermode coupling occurs through a set of elementary harmonic gratings that form the three-dimensional power spectrum of the film surface roughness for the optical waveguide $G(K)$, related by the Fourier transform with the field correlation function $f(r)$. The grating parameters are period $\Lambda$, grating vector $K = 2\pi N_p/\Lambda$ with normal $N_p$, and grating height $\delta$. By virtue of the optical linearity of the problem under study and statistical independence of the components $G(K)$, the waveguide modes are scattered by each grating independently of the remaining gratings. This allows us to calculate the total radiation field of waveguide modes as a superposition of fields of waveguide modes scattered by partial components $G(K)$. To this end, we now consider scattering of waveguide modes by a single diffraction grating representing an elementary harmonic perturbation $U(x,z)$ of the surface optical waveguide layer. As a result of scattering of waveguide modes by the component of the spatial frequency spectrum $S(K) = [G(K)]^{1/2}$ of the diffraction grating, the field of the corresponding radiation mode $E(r)$ with wave vector $k_0$ is formed. As is well known, such energy exchange is established only under condition of phase matching [1–4]. To consider the corresponding requirements imposed on the waveguide mode field and
diffraction grating, it is convenient to describe them by spatial frequency spectra \( E_s(k_s) \) and \( S(K) \) or angular spectra \( E_s(\theta_s) \) and \( S(\theta) \). In these spectra, the grating vector \( K_0 \) that meets the condition of phase matching is generally deviated from \( K_0 \) by the phase mismatch \( \Delta K_0 \). The angular dependences of vectors \( k_s(\theta_s) \), \( k(\theta) \), \( K(\psi) \), and \( \Delta K_0 \) and diffraction angle \( \theta \) can be easily found near phase matching by taking advantage of the orthogonality condition for the hodographs of angular dependences of vectors \( K(\psi) \) and \( k_s(\theta_s) \) to the optical waveguide surface:

\[
\Delta K(\psi, \theta_s) = \Delta K_0 + \left( k_{s0} \theta_s + \frac{K_{00}}{\cos \psi_0} \right) n, \tag{2}
\]

\[
\theta(\psi, \theta_s) = 0. \tag{3}
\]

To find the amplitudes of modes \( E_s(x, z) \) and \( E(x, z) \) in the rough layer of the optical waveguide, we now take advantage of the spectral approach. To this end, we describe the waveguide and radiation mode fields by their spectra \( E_s(k_s) \) and \( E(k) \). According to the method of coupled waves, we now consider the influence of perturbation \( U(x, z) \) only on the amplitudes of components \( \varphi_s(k_s) \) and \( \varphi(k) \). To this end, we approximately describe the waveguide mode profile \( E_s(x) \) and the modulus of its wave vector as well as polarization \( e \) and wave vectors of plane waves in the waveguide mode spectrum by wave equations for the nonperturbed medium. We note also that the gradient of distributions \( \varphi(x) \) and \( \varphi_s(k_s) \) can change only in the direction of waveguide mode propagation. In this case, the method of coupled waves yields the following truncated expressions for the spectral densities \( \varphi_s(k_s) \) and \( \varphi(k) \):

\[
\int_{\infty}^{+\infty} \int_{\infty}^{+\infty} \left( \frac{\partial \varphi_s(k_s)}{\partial z} \cos \psi + \frac{\partial \varphi_s(k_s)}{\partial x} \sin \psi \right) \exp(ik_s \cdot r) dk_s = -i\xi \left[ \int S(K) \varphi_s(k_s) \exp[-i(K + k_s) r] dK dK_s, \right.
\]

\[
\int_{\infty}^{+\infty} \int_{\infty}^{+\infty} \left( \frac{\partial \varphi_s(k_s)}{\partial z} \cos \psi + \frac{\partial \varphi_s(k_s)}{\partial x} \sin \psi \right) \exp(ik_s \cdot r) dk_s = -i\xi \left[ \int S(K) \varphi_s(k_s) \exp[i(K + k_s) r] dK dK_s, \right]
\]

where \( \xi = \frac{\pi(n_2^2 - n_1^2)}{\lambda_0} \) is the coupling coefficient, and \( \psi \) is the glancing angle of the waveguide mode. Taking advantage of Eqs. (2) and (3), we now transform the left part of the first equation of system (4):

\[
\int_{\infty}^{+\infty} \int_{\infty}^{+\infty} \left( \frac{\partial \varphi_s(k_s)}{\partial z} \cos \psi + \frac{\partial \varphi_s(k_s)}{\partial x} \sin \psi \right) \exp(ik_s \cdot r) dk_s = \frac{\partial E}{\partial z} \cos \psi + \frac{\partial E}{\partial x} \sin \psi \exp(ik_s \cdot r).
\]

The right side of this equation can be simplified by dividing it into \( \exp(k_0 \cdot r) \) and taking into account Eq. (2):

\[
\int_{\infty}^{+\infty} \int_{\infty}^{+\infty} S(\psi) \varphi_s(\theta_s) \exp[-i\Delta K(\theta_s, \psi) r] d\psi d\theta_s = \exp[-i\Delta K_0 x] \int_{\infty}^{+\infty} S(\psi) \exp \left[ -i \left( \frac{K_{00}}{\cos \psi_0} \right) X \right] \int_{\infty}^{+\infty} \varphi_s(\theta_s) \exp \left[ -i \left( |k_{s0}| \theta_s \right) X \right] d\psi d\theta_s = U(K^*) E_s(x, z) \exp[-i\Delta K_0 x].
\]

Taking into account that the left side of the second equation of system (4) is the spectrum of the waveguide mode profile gradient, we divide it into \( \exp(k_0 \cdot r) \). As a result, we obtain

\[
\int_{\infty}^{+\infty} \int_{\infty}^{+\infty} S(\psi) \varphi_s(\theta_s) \exp[i\Delta K(\psi, \theta_s) x] d\psi d\theta_s = E \exp[i\Delta K_0 x] \int_{\infty}^{+\infty} S(\psi) \exp[i\Delta K(\psi) x] d\psi.
\]