A spinor field interacting with the Aaronov–Bohm external field is examined. Analytical expressions for the vacuum average components of the energy-momentum tensor are derived. Dependences of the components of energy-momentum tensor of the spinor field in the vacuum state on the distance and field strength are investigated.

INTRODUCTION

The present work is a direct continuation of [1] and is devoted to calculations of the vacuum average components of the energy-momentum tensor (EMT) of a spinor field in the Aaronov–Bohm classical field. Analogous problem was solved in [1] for a charged scalar field, and the existence of vacuum effects caused by nontrivial space topology was demonstrated. The principal difference of the situation examined here is that the examined quantum field possesses a spin that should affect the character of the vacuum effects. The present study focuses on the influence of the spin on the vacuum quantum effects and, in particular, the polarization of vacuum caused by the presence of the Aaronov–Bohm classical field and actually by the nontrivial space topology.

In all calculations, we used the system of units in which $c = 1$, $\hbar = 1$, and the metric had the signature (+,−,−,−). The spatial part of the 4-potential of the electromagnetic field satisfies the Coulomb gauge relation $\nabla A = 0$. The Greek indices run over values 0, 1, 2, and 3 and as usual, summation is carried out over the recurrent indices.

ENERGY-MOMENTUM TENSOR OF A SPINOR FIELD

1. We now consider the interaction of the spinor field with the Aaronov–Bohm classical field. The 4-potential vector of the Aaronov–Bohm field in cylindrical coordinates ($x^0 = t$, $x^1 = r$, $x^2 = \varphi$, and $x^3 = z$) has the form

$$A^\mu = \left(0, \frac{a}{r}, 0, 0\right).$$

Here $a = \frac{1}{2} BR^2$, $B$ is the magnetic field induction inside a solenoid, and $R$ is the solenoid radius. The density of the spinor field Lagrangian in the classical external field has the form

$$L(x) = \frac{i}{2} \bar{\Psi}(x)\gamma^\mu (D_\mu \Psi(x)) - (D^\mu \bar{\Psi}(x))\gamma_\mu \Psi(x) - m \bar{\Psi}(x)\Psi(x),$$

where $\gamma^\mu$ are the Dirac matrices, $D_\mu \Psi(x)$ is the covariant derivative of the spinor field, and $m$ is the mass of the spinor field.

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where $\Psi$ and $\bar{\Psi}$ are four-component field functions whose components form bispinors (the bar atop the field function denotes the Dirac conjugation), and $m$ is the mass corresponding to the given field. The field equations

$$
\left( \gamma^\mu D_\mu - m \right) \Psi(x) = 0, \quad \bar{\Psi}(x) \left( \gamma^\mu D_\mu^* + m \right) = 0 \tag{3}
$$

correspond to Lagrangian (2). In this case, the energy-momentum tensor assumes the form

$$
T_{\mu\nu} = \frac{i}{2} \left[ \bar{\Psi}(x) \gamma_\mu \Psi(x) - (\gamma_\nu \bar{\Psi}(x)) \gamma_\mu \Psi(x) \right].
$$

Equations (3) in field (1) admit complete separation of variables. The system of finite solutions for the given equation has the form

$$
\psi_{n}^{(\pm)}(k, \lambda; r, \varphi, z, t) = \exp(-i[\varepsilon \omega t - n\varphi - kz]) \begin{cases}
C_1 J_{\alpha_1}(\lambda r) e^{-i\varphi} \\
C_2 J_{\alpha_2}(\lambda r) e^{i\varphi} \\
C_3 J_{\alpha_1}(\lambda r) e^{-i\varphi} \\
C_4 J_{\alpha_2}(\lambda r)
\end{cases},
$$

where $\omega^2 = \lambda^2 + k^2 + m^2$, $\alpha_1 = n - ea - 1$, $\alpha_2 = n - ea$, $\varepsilon$ defines the solution frequency factor ($\varepsilon = -1$ specifies solutions with negative frequencies, and $\varepsilon = +1$ specifies solutions with positive frequencies). Integration constants $C_{1,2,3,4}$ satisfy the following relations:

$$
C_3 = \frac{kC_1 - i\lambda C_2}{\varepsilon\omega + m}, \quad C_4 = \frac{i\lambda C_1 - kC_2}{\varepsilon\omega + m}, \quad C_1^* C_1 + C_2^* C_2 = \frac{\lambda(\varepsilon\omega + m)^2}{8\pi^2 \varepsilon(\omega + \varepsilon m)}.
$$

Considering that the Bessel functions form the complete orthogonal system with the following orthogonality and completeness conditions [2]:

$$
\int_{0}^{\infty} J_{\alpha}(\lambda r) J_{\alpha'}(\lambda' r) r dr = \frac{1}{\lambda} \delta(\lambda - \lambda'), \quad \int_{0}^{\infty} J_{\alpha}(\lambda r) J_{\alpha'}(\lambda' r) \lambda d\lambda = \frac{1}{r} \delta(r - r'),
$$

where $\lambda > 0$, $\lambda' > 0$, $r > 0$, and $r' > 0$, we can easily demonstrate by direct substitution that the system of solutions obtained is complete. Thus, general solutions (3) assume the form

$$
\Psi(r, \varphi, z, t) = \sum_{n=-\infty}^{\infty} \int d\lambda [a_n(k, \lambda) \Psi_n^{(+)}(k, \lambda) + b_n^{+}(k, \lambda) \Psi_n^{(-)}(k, \lambda)],
$$
$$
\bar{\Psi}(r, \varphi, z, t) = \sum_{n=-\infty}^{\infty} \int d\lambda [a_n^{+}(k, \lambda) \bar{\Psi}_n^{(+)}(k, \lambda) + b_n(k, \lambda) \bar{\Psi}_n^{(-)}(k, \lambda)].
$$

2. The quantization procedure involves transition to operators $\Psi, \Psi^+ \rightarrow \hat{\Psi}, \hat{\Psi}^+$ with the following commutation rules:

$$
\{\Psi(r, t), i\Psi^+(r', t)\} = i\delta(r - r'),
$$

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