RENORMALIZATION BEYOND THE FRAMEWORK OF THE PERTURBATION THEORY

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By the example of a scalar field model with $\phi^4$ interaction, the possibility of separating the finite parameters beyond the framework of standard (in the interaction representation) perturbation theory is discussed. A scheme of divergence separation for large momenta and their compensation by infinite initial parameters – mass and coupling constant – is examined. The one-particle sector of the diagonal component of the total Hamiltonian admits this procedure. In this case, the energy of the one-particle spectrum has a relativistic form.

Keywords: quantum field theory, scalar field, renormalization, Hamiltonian.

INTRODUCTION

Investigation of relativistic quantum field theory with local interaction has revealed the necessity of separating finite values of the parameters from infinite bare mass, coupling constants, etc. Such procedure is called renormalization and is fulfilled in the context of the perturbation theory (for example, see [1] and the references therein). A study of nonrelativistic theoretical field models with contact interaction [2] on the one hand, faces a similar problem of determination of finite parameters and on another hand, allows calculations to be performed beyond the framework of the perturbation theory. It is of interest to apply the methods developed for a description of nonrelativistic models to relativistic quantum field theory.

The description of nonrelativistic models [2–5] is based on the problem of eigenvalues for total Hamiltonians. This problem makes sense, since by virtue of nonrelativistic character of the interaction, the number of particles is the integral of motion and hence, the operator of the number of particles commutes with the total Hamiltonian. Not addressing to the strict mathematical theory of constructing the self-adjoint operators in the Hilbert space, we note that one of such extensions in nonrelativistic models can be constructed using the cutoff procedure for large momenta or in short, $\Lambda$-cutoff [5]. Such scheme of separating the finite parameters can also be used for a description of relativistic models.

In this work, a scalar field model with quadrupole interaction is considered as an illustration. The emphasis is on the possibility of renormalization of model parameters outside of the framework of the perturbation theory. The brief scheme of construction of model solution is presented in [6]. It is based on the representation of the total Hamiltonian by the sum of two components (compare with the interaction representation). For one term called diagonal it is possible to construct, by analogy with the nonrelativistic model, a complete set of eigenfunctions and to determine the procedure of separation of finite parameters. The second term – the fluctuation component – is taken into account as a perturbation that needs no additional renormalization.
Let us write down the Hamiltonian of the model in the form

\[
H = \frac{1}{2} \int d^3x \left[ \phi^2 + (\nabla \phi)^2 + m_0^2 \phi^2 - \frac{\lambda}{2} \phi^4 \right].
\]  

(1)

The choice of the model is determined by several reasons. First of all, due to its apparent simplicity, the scalar field model is used as a certain theoretical laboratory for an analysis of various methods and approaches in the quantum field theory. Furthermore, the results of analysis do not require experimental support, leaving the possibility of speculative constructions. For example, the sign of the constant \( \lambda \) differs from the conventional one (see any book devoted to the quantum field theory) for which the Hamiltonian is a bounded operator. It is expedient to note that the chosen sign of coupling constant (1) provides the existence of bound states [6].

We now consider the fields included in \( H \) at \( t = 0 \) (the Schrödinger representation). The motivation of this choice is connected with the statement of the problem on the eigenvalue for the diagonal Hamiltonian component. For the field operators, we choose representation

\[
\phi(x,0) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\sqrt{2E(k)}} \left[ a(k)e^{ikx} + a^+(k)e^{-ikx} \right],
\]

\[
\phi(x,t)\big|_{t=0} = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3k}{\sqrt{2E(k)}} E(k) \left[ -a(k)e^{ikx} + a^+(k)e^{-ikx} \right] = \Pi_0(x,0).
\]  

(2)

Here \( \Pi_0(x,0) \) is the canonical momentum of the field \( \phi(x,0) \), \([\phi(x,0),\Pi(y,0)] = i\delta(x - y)\), or for \( a(k), a^+(k) : [a(k), a^+(q)] = \delta(k - q) \). An arbitrary parameter \( E(k) \) can be expressed through the parameter of the Bogolyubov transformation

\[
b^+(k) = \cosh \theta(k)a^+(k) + \sinh \theta(k)a(-k),
\]

\[
b(k) = \cosh \theta(k)a(k) + \sinh \theta(k)a^+(-k),
\]

\[
4\theta(k) = \ln \frac{k^2 + m_0^2}{E^2(k)} \equiv \ln \frac{E^2_0(k)}{E^2(k)},
\]  

(3)

which belongs to the class of unitary nonequivalent transformations. The operators \( b(k) \) and \( b^+(k) \) are determined by the free component \( H_0 \) of the total Hamiltonian.

Substituting \( \phi(x,0) \) and \( \phi(x,0) \) from Eq. (2) into Hamiltonian (1) and determining diverging integrals as functions of \( \Lambda \) (the cutoff parameter)

\[
\frac{1}{V} = \frac{1}{(2\pi)^3} \int_0^\Lambda \frac{d^3k}{2E(k)},
\]

\[
\left\langle k^{2n} \right\rangle = \frac{V^*}{(2\pi)^3} \int_0^\Lambda \frac{d^3k}{2E(k)} k^{2n},
\]  

(4)

we obtain for the \( H \) representation