ELEMENTARY PARTICLE PHYSICS AND FIELD THEORY

SOME ASTROPHYSICAL EFFECTS OF THE FIVE-DIMENSIONAL GEOMETRICAL THEORY OF GRAVITATION AND ELECTROMAGNETISM

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As is well known, the Einstein equations within the monade formalism [1] in empty five-dimensional space-time with additional spatial dimension \( x^4 \)

\[ R_{AB} - \frac{1}{2} R g_{AB} = 0 \quad (A, B = 0, 1, 2, 3, 4) \]  

are separated into the combined system of the Maxwell–Einstein equations in the four-dimensional Riemannian space-time. This is obtained under condition that the metric coefficients \( g_{AB} \) are independent of the additional spatial coordinate \( x^4 \) (the cylindricity condition).

In this case, the additional nondiagonal metric coefficients \( g_{4k} \) \((k = 0, 1, 2, 3)\) appear proportional to the components of the electromagnetic 4-vector potential \( A_k \), and the metric coefficient \( g_{44}(x^k) \) corresponds to the presence of a certain scalar field of geometrical origin.

The cylindricity condition for the fifth coordinate from the general group of coordinate transformations

\[ x'^k = x^k + f(x^4), \quad x'^4 = x^4 + f(x^4). \]  

Here \( x^k \) and \( x^4 \) are the coordinates of the four-dimensional space-time.

In the present work, within the limits of the geometrical five-dimensional theory of gravitation and electromagnetism indicated above, we study consequences of this theory in the presence of an azimuth geometrized magnetic field and a scalar geometrized field \( g_{44} = F(x) \) in the stationary case. This problem can be more conveniently solved in a space having cylindrical symmetry.

This problem of gravitational interaction of the azimuth magnetic field has already been considered by K. A. Bronnikov [2] within the framework of general relativity theory (GRT). The purpose of the present work is to obtain results within the framework of geometrical approach to this problem and to consider the influence of other geometrical objects that can be present in the examined theory.

We take the five-dimensional cylindrically symmetric metric for the examined problem in the form

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\[ ds^2 = -A(x)dx^2 + B(x)dx^2 + C(x)d\alpha^2 + D(x)dz^2 + 2E(x)dxdy + F(x)(dx^4)^2. \]  

Here the metric coefficient \( g_{44} = F(x) \) describes the scalar field of geometrical origin, and the coefficient \( g_{34} = E(x) \) corresponds to the component of the magnetic potential, so that its derivative with respect to the radial coordinate \( x \) is proportional to azimuth magnetic field strength \( H_\alpha : H_\alpha = \frac{E'}{2\sqrt{\chi}}, \) where \( \chi = \frac{8\pi G}{c^4}. \)

The Einstein vacuum equations in the space-time with metrics (3) assume the form

\[
\frac{C''}{C} + \frac{\Delta'}{\Delta} - \frac{C^2}{2\Delta^2} - \frac{\Delta'^2}{2\Delta^2} - \frac{B'\Delta'}{2B\Delta} + \frac{C'\Delta'}{2C\Delta} - \frac{B'C'}{2BC} - \frac{D'F' - E'^2}{2\Delta} = 0,
\]

\[
\frac{A'C'}{AC} + \frac{A'\Delta'}{A\Delta} + \frac{C'\Delta'}{C\Delta} + \frac{D'F' - E'^2}{\Delta} = 0,
\]

\[
\frac{A' + \Delta'}{A} - \frac{A^2}{2A^2} - \frac{\Delta^2}{2\Delta^2} - \frac{B'\Delta'}{2B\Delta} + \frac{A'B'}{2AB} + \frac{A'C'}{2AC} + \frac{A'F'}{2AF} + \frac{C'F'}{2CF} - \frac{B'F'}{2BF} - \frac{F'\Delta'}{2F\Delta} + \frac{D'F' - E'^2}{2\Delta} = 0,
\]

\[
\frac{A'' + \Delta'}{A} - \frac{A^2}{2A^2} - \frac{\Delta^2}{2\Delta^2} - \frac{B'\Delta'}{2B\Delta} + \frac{A'B'}{2AB} + \frac{A'C'}{2AC} + \frac{A'E'}{2AE} + \frac{C'E'}{2CE} - \frac{B'E'}{2BE} - \frac{E'\Delta'}{2E\Delta} + \frac{D'F' - E'^2}{2\Delta} = 0,
\]

where \( \Delta = DF - E^2. \)

To solve this system, we take advantage of the harmonic coordinates:

\[ B = A\Delta, \quad \frac{B'}{B} = \frac{A'}{A} + \frac{C'}{C} + \frac{\Delta'}{\Delta}. \]  

Let us consider first the problem with the geometrical scalar field alone (\( E(x) = 0 \) and \( F(x) \neq 1 \)). This problem with spherical symmetry has already been solved in [3].

The general solution of system of equations (4) for the examined case under condition (5) is the following:

\[ A(x) = e^{(m+b)x}, \quad C(x) = e^{bx}, \quad D(x) = e^{(b+k)x}, \quad B(x) = e^{(4b+2k+m)x}, \quad F(x) = e^{(b+k+a)x}, \]  

where the integration constants \( a, b, k, \) and \( m \) are related by the condition

\[ (m+b)(3b+2k+a) + (b+k)(3b+k+a) + ab = 0. \]  

By choosing constants \( a, b, k, \) and \( m \) with allowance for condition (7), we can derive a solution with plane or string asymptote. In this case, the conditions for this asymptote are the following [2]:

\[ m+b = 0, \quad b = 2, \quad \frac{C'}{4BC} \to 1 - \frac{1}{\xi}, \quad \text{for} \ x \to \infty, \]