ON THE POSSIBLE EXISTENCE OF WORMHOLES WITHOUT GRAVITATIONAL FORCES

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A study of the existence of “wormholes” and pathways of their formation, which is a problem of current interest, is continued. Of especial interest is the possible existence of traversable “wormholes,” in particular those in which gravitational forces do not act on test particles. It is shown that in the presence of a vortex gravitational field interacting with self-gravitating physical fields, such conditions can be met.

Keywords: vortex gravitational field, traversability of wormholes, rotating ideal gas, astrophysics, nonlinear scalar field, gravitational forces.

Amongst the big questions in astrophysics today, one of the most interesting is the question of the existence of wormholes and the possibility of their realization.

An important part of this problem is the task of obtaining traversable wormholes in which there are no singular regions, horizons, or very strong gravitational forces hindering the motion of test particles.

In this context, let us discuss the question of the existence of wormholes in which there are no gravitational forces at all acting on the test particle.

As is well known [1], a gravitational force \( F_g \) in the Riemannian space within the framework of GTR is given by the formula

\[
F_g = \frac{mc^2}{\sqrt{1 - \frac{\omega^2}{c^2}}} \left( -\text{grad} \ln g_{tt} + \sqrt{g_{tt}} \left[ \frac{\mathbf{v}}{c} \times \mathbf{\omega} \right] \right).
\]

(1)

Here \( m \) and \( \mathbf{v} \) are the mass and velocity of the test particle, \( g_{tt} \) is the metric coefficient of \( dt^2 \) in the metric, and \( \mathbf{\omega} \) is the angular velocity of rotation of the reference frame, expressed in covariant form by the formula

\[
\omega^i = \frac{1}{2} \varepsilon^{iklm} \tau_k \tau_{l,m},
\]

(2)

where \( \tau_k \) is the unit tangent vector to the coordinate time lines of the reference frame. The vector \( \omega^i \), as can be seen from formula (2), is also equal to the angular velocity of rotation of congruences of time lines and is a kinematic characteristic of a vortex gravitational field [2, 3].

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From formula (1) it is clear that when \( g_{tt} = 1 \) the gravitational force acting on a test particle at rest will be equal to zero: \( F_g = 0 \). If, however, \( \nu \neq 0 \), then only the Coriolis force defined by the second term in formula (1) will act on it.

Below we show that the condition \( g_{tt} = 1 \) is possible only in wormholes formed by a vortex gravitational field interacting with self-gravitating matter in the form of a rotating ideal liquid or with a nonlinear scalar field with potential \( V(\phi) \).

To start with, let us consider stationary configurations of a self-gravitating rotating ideal liquid in which the gravitational field satisfies the condition \( g_{tt} = 1 \) over the entire region, i.e., the condition of the absence of a gravitational force acting on the test particle.

The simplest stationary metric consistent with a self-gravitating rotating ideal liquid is a cylindrically symmetric metric of the form

\[
ds^2 = e^\lambda dx^2 + e^\beta d\phi^2 + e^\alpha dz^2 - e^\gamma dt^2 + 2e^\gamma dt d\phi.
\]

Here \( g_{tt} = e^\nu \) and all the metric coefficients depend only on the radial coordinate \( x \).

The metric of a spacelike surface of 4-dimensional spacetime (3) is given by the expression

\[
dl^2 = e^\lambda dx^2 + e^\mu d\phi^2 + e^\alpha dz^2,
\]

where the angular metric coefficient \( e^\mu \) is defined by the formula

\[
e^\mu = \frac{e^{\beta+\nu} + e^{2\gamma}}{e^\gamma}.
\]

Variation of this metric coefficient defines a point on the symmetry axis \( oz \) and spatial infinity: when \( e^\mu = 0 \), we have the symmetry axis, and as \( e^\mu \to \infty \) we obtain spatial infinity.

In a space with metric (3) there exists a stationary vortex field for which the angular velocity of rotation in accordance with formula (2) is given by the expression

\[
\omega = \frac{\omega_0}{2} e^{-\frac{(\nu+\frac{1}{2})}{2}} \quad (\omega_0 = \text{const}).
\]

Here the constant \( \omega_0 \) defines some boundary condition for the angular velocity, where \( 2e\frac{\nu+\frac{1}{2}}{2} = 1 \).

To start with, we shall use the so-called isothermal coordinates, where \( e^\lambda = e^{\alpha} \), for which the physical consequences of the obtained solutions of the Einstein equations are more clearly visible, since for flat space the metric automatically satisfies this coordinate condition:

\[
ds^2 = dr^2 + r^2 d\alpha^2 + dz^2 - dt^2,
\]

i.e., we have \( e^\lambda = e^{\alpha} = 1 \).

The system of gravitational Einstein equations for a self-gravitating rotating ideal liquid for the case under consideration with formula (6) taken into account has the form

\[
\lambda', \mu' + \lambda' \nu' + \mu' \nu' = 4pe^{\lambda} - 4\omega_0^2 e^{-2\nu},
\]