ERRORS DUE TO THE ANISOTROPIC-COMMON-RAY APPROXIMATION OF THE COUPLING RAY THEORY

L. KLIMŠE AND P. BULANT

Department of Geophysics, Faculty of Mathematics and Physics, Charles University,
Ke Karlovu 3, 121 16 Praha 2, Czech Republic

Received: October 7, 2005; Revised: February 30, 2006; Accepted: April 9, 2006

ABSTRACT

The common-ray approximation eliminates problems with ray tracing through S-wave singularities and also considerably simplifies the numerical algorithm of the coupling ray theory for S waves, but may introduce errors in travel times due to the perturbation from the common reference ray. These travel-time errors can deteriorate the coupling-ray-theory solution at high frequencies. It is thus of principal importance for numerical applications to estimate the errors due to the common-ray approximation applied.

The anisotropic-common-ray approximation of the coupling ray theory is more accurate than the isotropic-common-ray approximation. We derive the equations for estimating the travel-time errors due to the anisotropic-common-ray (and also isotropic-common-ray) approximation of the coupling ray theory. The errors of the common-ray approximations are calculated along the anisotropic common rays in smooth velocity models without interfaces. The derivation is based on the general equations for the second-order perturbations of travel time.

Key words: coupling ray theory, common-ray approximation, travel time, perturbation theory, seismic anisotropy, heterogeneous media

1. INTRODUCTION

There are two different high-frequency asymptotic ray theories: the isotropic ray theory based on the assumption of equal velocities of both S waves, and the anisotropic ray theory assuming both S waves strictly decoupled. In the isotropic ray theory, the S-wave polarization vectors do not rotate about the ray, whereas in the anisotropic ray theory they coincide with the eigenvectors of the Christoffel matrix which may rotate rapidly about the ray. Thomson et al. (1992) demonstrated analytically that the high-frequency asymptotic error of the anisotropic-ray-theory wavefield is inversely proportional to the second or higher root of the frequency if a
ray passes through the point of equal S-wave eigenvalues of the Christoffel matrix even in an otherwise strongly anisotropic medium.

In “weakly anisotropic” models, at moderate frequencies, the S-wave polarization tends to remain unrotated round the ray but is partly attracted by the rotation of the eigenvectors of the Christoffel matrix. The intensity of the attraction increases with frequency. This behaviour of the S-wave polarization is described by the coupling ray theory proposed by Coates and Chapman (1998). The frequency-dependent coupling ray theory is the generalization of both the zero-order isotropic and anisotropic ray theories and provides continuous transition between them. The coupling ray theory is applicable to S waves at all degrees of anisotropy, from isotropic to considerably anisotropic velocity models. The numerical algorithm for calculating the frequency-dependent complex-valued S-wave polarization vectors of the coupling ray theory has been designed by Bulant and Klimáš (2002).

There are many commonly used quasi-isotropic approximations of the coupling ray theory (e.g., Pšenčík, 1998), which diminish the accuracy of the coupling ray theory both with increasing frequency and increasing degree of anisotropy. Refer to Bulant and Klimáš (2002, 2004) and to Klimáš and Bulant (2004) for the description of individual quasi-isotropic approximations and for the examples of their impact on synthetic seismograms. Most of these quasi-isotropic approximations can be avoided with minimal effort except for the common-ray approximation for S waves.

In the common-ray approximation, only one reference ray is traced for both anisotropic-ray-theory S waves, and both S-wave anisotropic-ray-theory travel times are approximated by the perturbation expansion from the common reference ray. Whereas tracing the continuous system of anisotropic-ray-theory rays may be very difficult in the vicinity of an S-wave singularity at which the S-wave slowness surfaces coincide (Vavryčuk, 2003), this problem does not occur in the common-ray approximation. The common-ray approximation thus eliminates problems with ray tracing through S-wave singularities and also considerably simplifies coding of the coupling ray theory and numerical calculations, but may introduce errors in travel times due to the perturbation. These travel-time errors can deteriorate the coupling-ray-theory solution at high frequencies. It is thus of principal importance for numerical applications to estimate the travel-time errors due to the common-ray approximation, and then the related error of the wavefield.

In the common-ray approximation, the S-wave travel times are usually approximated by the first-order perturbation expansion from the common reference ray. The errors of S-wave travel times may then be approximated by the second-order terms in the perturbation expansion. Note that the perturbation expansion is the Taylor expansion with respect to the perturbation parameters, see (19), which parametrize the Hamiltonian, see (18). We refer here to the partial derivatives with respect to the perturbation parameters as perturbation derivatives, in order to distinguish them from the partial derivatives with respect to the spatial coordinates.

The common reference rays have routinely been represented by the isotropic common rays calculated in the isotropic reference model, but they would better be represented by the anisotropic common rays. The anisotropic common rays are