CONSTITUTIVE EQUATIONS OF STRAIN-HARDENING THEORY
FOR NONISOTHERMAL DEFORMATION PROCESSES

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A variant of the strain-hardening theory is proposed for describing nonisothermal deformation processes. The author postulates the dependence of parameters of the constitutive equations on stress and temperature. The influence of the loading history on the creep rate variation is allowed for by means of a scalar function of damage level. A procedure for more specific definition of the constitutive relationships is outlined. Efficiency of this approach is illustrated by describing creep curves for chromium-nickel steels over a fairly wide range of variation of stresses and temperature.

Keywords: nonisothermal deformation processes, creep, constitutive equations, prolonged fracture model, damage level, instantiation of equations.

Introduction. The importance of studies of high-temperature deformation and fracture processes under long-term loading depends primarily on solving practical problems arising during the design of steam and gas turbines, jet and spacecraft engines, high-speed aircraft, nuclear reactors, etc. Structural elements operate under unsteady heating conditions that greatly affect physico-mechanical properties of materials and give rise to considerable temperature gradients. The necessity of predicting the above-mentioned deformation processes and assessing the bearing strength of structures in the load cases close to actual operating conditions has given impetus to extensive experimental studies. They have resulted in the elaboration of numerous research techniques and development of the required experimental equipment and machines.

At present, a great body of experimental data on high-temperature deformation and fracture in materials has been accumulated. Specifically, when subjected to the above-mentioned loading conditions the structures experience the phenomena of static and cyclic creep, stress relaxation in and aging of material, low- and high-cycle fatigue. Also, the effect of strain rate on the stress distribution in space and time becomes more intensive, the deformable material undergoes structural degradation, the microstructure, surface state and various impurities in material have a considerable influence on creep, there occurs nucleation and propagation of microcracks, voids, cracks, etc.

The experimental findings enabled researchers to get a deeper insight into the deformation and fracture processes, reveal and explain some new qualitative effects. They provide input information for setting up constitutive equations of inelastic deformation theories and prolonged-fracture models. On the strength of these data, researchers formulate starting hypotheses for deformation and fracture models, determine parameters for constitutive equations, and assess reliability of the theories elaborated. The results of such studies have been most comprehensively reviewed in [1–18].

Analysis of Constitutive Equations Describing Creep Processes in Materials. The notion of strain, specifically the creep strain, is taken as the principal characteristic in the theory of materials. As mentioned in [15], for structural metals and alloys the strains that infinitely develop with time at rather high temperatures are usually considered to mean the creep strains proper.

Restricting our consideration to a one-dimensional case we assume that the total deformation \( \varepsilon \) can be written as
\[
\varepsilon = \psi(\sigma) + \varepsilon^c + \alpha_T (T - T_0),
\]

where \(\psi(\sigma)\) is the instantaneous deformation including an elastic \((\varepsilon^e)\) and plastic \((\varepsilon^p)\) components, \(\sigma\) is the stress, \(\varepsilon^c\) is the creep strain, \(\alpha_T\) is the linear expansion coefficient, and \(\alpha_T (T - T_0)\) is the thermal deformation.

In engineering practice, the ageing, flow, and strain-hardening theories are most commonly used for describing creep processes. The respective constitutive equations can be represented as

\[
\varepsilon = \Phi_1 (\sigma, T, t),
\]

\[
\varepsilon^c = \Phi_2 (\sigma, T, t),
\]

\[
\varepsilon^c = \Phi_3 (\sigma, T, \varepsilon^c),
\]

where \(t\) is time, \(\Phi_1, \Phi_2,\) and \(\Phi_3\) are power or exponential functions; a dot above a symbol denotes differentiation with respect to time.

Each of the above-mentioned theories can describe only one of three characteristic segments of the creep curve. The theory of ageing and strain hardening represents nonsteady creep, while the flow theory describes merely the second steady segment of the creep curve. Their main drawback is that they ignore the effect of loading history on the current stress-strain state of material, which is confirmed by numerous experimental data. Note that this applies also to other theories that involve constitutive equations of differential type.

The hereditary theory of solids (linear and nonlinear viscoelastic theories) offers a somewhat larger potential for describing creep processes [2]. Its integro-differential constitutive equations account for the influence of loading history on inelastic deformation of a media and qualitatively describe mechanical behavior of actual solids. The constitutive equations turned out to be quite convenient and reliable when used for the description of deformation of rock, concrete, polymeric materials, and reinforced plastics. However, the application of the equations to describe mechanical behavior of structural steels and alloys results in considerable errors.

The most general rules of deformation are described by the endochronic theory of plasticity [16, 19] and the theory of elastoplastic processes [20]. The first-mentioned theory allows for the effect of the overall loading history on the deformation process, while the second one takes into consideration only a part of it. However, being inadequately elaborated, lacking the necessary mathematical software, and requiring most cumbersome computations, these theories are currently unsuitable for the calculation of particular structural elements.

An approach that generalizes relationships (2)–(4) on the basis of the concept of “latent” or “internal” variables is believed to hold much promise. The defining relationships involved describe mechanical behavior of a fairly wide class of materials, yet are very convenient and simple. The internal variables enable one to take into account the influence of loading history and temperature on the current stress-strain state. According to the hypothesis of parameters of state the thermodynamic condition of inelastic media is described by a strain tensor, temperature, temperature gradient, and a finite or infinite set of internal variables. These variables are governed by some set of differential or integro-differential evolutionary equations and can be scalar, vector or tensor quantities [21].

We neglect the temperature gradient and restrict our consideration to a single internal variable in the differential-type constitutive equations of the general thermodynamic theory and thus on the basis of representation (1) we have

\[
\dot{\varepsilon}^c = f_1 (\sigma, \varepsilon^c, \omega, T),
\]

\[
\dot{\omega} = f_2 (\sigma, \varepsilon^c, \omega, T),
\]

where \(\omega\) is the scalar-type variable; \(f_1\) and \(f_2\) are the functions to be determined.