Abstract. Is standard, Kripke-style semantics for quantified modal logic compatible with the view that no individual may belong to more than one possible world? One is naturally inclined to answer in the negative, arguing that the view requires a Lewis-style, counterpart-theoretic semantics instead. Strictly speaking, however, this natural answer is wrong-headed. This note explains why.

Keywords: Trans-World Identity, Counterpart Theory, Kripke, Lewis, Semantics vs. Metaphysics.

It is natural to think that a standard, Kripke-style semantics for quantified modal logic (QML) is incompatible with the view that no individual can exist in more than one possible world, a view that seems to require a Lewis-style, counterpart-theoretic semantics instead. Strictly speaking, however, this thought is wrong-headed. A standard semantics regards a modal statement such as ‘I might have been fat’ as true only if I am in the extension of ‘is fat’ at some other possible world, whereas counterpart theory regards it as true only if a counterpart of mine is in the extension of ‘is fat’. But just as the truth conditions of counterpart theory are in principle compatible with the possibility (rejected by Lewis) that some individuals qualify as their own other-worldly counterparts, the truth conditions of a standard semantics are in principle compatible with the possibility (dismissed by Kripke) that all individuals are world-bound. Here is how.

1. Consider a standard way of setting up the semantics. This involves two tasks. First, we have to say what sorts of structures qualify as models of the language, and then we have to spell out a corresponding definition of truth for all formulas of the language. Concerning the first task, the basic idea is that a model $M$ must specify the following four ingredients:

- a non-empty set $W$ of possible worlds;
- an accessibility relation $R$ on $W$;
- a function $D$ assigning to each world $w \in W$ a non-empty domain of individuals $D(w)$;
— a function $V$ assigning to each variable $x$ an individual $V(x)$ and to each $n$-ary predicate $P$ an extension $V(P)(w)$ for each $w \in W$.\footnote{If the language contains names, $V$ will also assign values to each name; to keep things simple, however, I shall assume the non-modal vocabulary to be that of pure quantification theory.}

On this basis, the second task is cashed out in terms of a recursive definition of the conditions under which a formula $\phi$ is true at a world $w$ in a model $M$, written $\models^M_w \phi$. For instance, with the usual connectives and quantifiers the definition goes like this:

- $\models^M_w P x_1 \ldots x_n$ iff $(V(x_1), \ldots, V(x_n)) \in V(P)(w)$;
- $\models^M_w \neg \phi$ iff not $\models^M_w \phi$;
- $\models^M_w \phi \land \psi$ iff $\models^M_w \phi$ and $\models^M_w \psi$;
- $\models^M_w \diamond \phi$ iff $\models^M_{w'} \phi$ for some $w'$ such that $wRw'$;
- $\models^M_w \Box \phi$ iff $\models^M_{w''} \phi$ for every $w''$ such that $wRw''$;
- $\models^M_w \exists x \phi$ iff $\models^M_{w'} \phi$ for some $x$-variant $M'$ of $M$ such that $V'(x) \in D(w)$.
- $\models^M_w \forall x \phi$ iff $\models^M_{w'} \phi$ for every $x$-variant $M'$ of $M$ such that $V'(x) \in D(w)$.

(where an $x$-variant of a model $M = \langle W, R, D, V \rangle$ is any model $M' = \langle W', R', D', V' \rangle$ that agrees with $M$ on everything except possibly that $V'(x) \neq V(x)$).

Of course, in order for this picture to qualify as a general semantic framework, it must be neutral with respect to what modal principles will qualify as valid. That is, it must only validate those formulas of QML that capture the logic of $\diamond$ and $\Box$, as opposed to formulas that reflect controversial modal tenets. The distinction is not clear-cut, but it is intuitive enough to provide a criterion for testing the framework. For example, formulas such as

\[
\text{K} \quad \Box(\phi \rightarrow \psi) \rightarrow (\Box \phi \rightarrow \Box \psi)
\]
\[
\text{C} \quad (\Box \forall x \phi \land \Box \forall x \psi) \rightarrow \Box \forall x (\phi \land \psi)
\]

are usually regarded as expressing logical truths, whereas propositional principles such as

\[
\text{T} \quad \Box \phi \rightarrow \phi
\]
\[
\text{5} \quad \diamond \phi \rightarrow \Box \diamond \phi
\]