
Inspired by the book Algorithmics written by David Harel (Addison-Wesley, 1987; 1992; 2004) which "compares programming languages by showing the same little program in each language that was treated" (p. 1), the editor tries to do something similar for proof assistants. The result is this book which contains the proof of the theorem that $\sqrt{2}$ is irrational in 17 contemporary proof systems. It also provides an interesting foreword by Dana Scott which discusses the challenge(s) of computer-aided proofs in a general perspective.

The central idea of the book is to provide a comparison—somehow on the surface—of different proof systems. To this aim the editor chooses a theorem which shall serve as common example. As he points out in the introduction, "[t]here are two canonical proofs that are always used to show non-mathematicians what mathematical proof is: – The proof that there are infinitely many prime numbers. – The proof of the irrationality of the square root of two. From those two I selected the second, because it involves the real numbers." (p. 2). We think that this was a proper choice. Using this rather elementary example the book is accessible to everybody with elementary school knowledge of mathematics.¹ The editor does not ask for one particular proof of this theorem, but "for a proof that was 'typical' for the system, that would show off how the system was meant to be used".

The presentation of the different theorem provers follows a common scheme:

1. Statement
2. Definitions
3. Proof
4. System

The statement is the final statement of the proof in the syntax of the proof system; the subsection of definitions contains some sample definitions taken from the formalization or from the proof systems libraries. Both are extracted from the proofs by the editor, "to highlight the syntax of statements, and the syntax of definitions" (p. 4). The core of each presentation is the third part which contains the "formalization, typeset as closely as possible as it appears in the files that people sent me" (p. 4).²

The section about the system is organized as a questionnaire, containing the answers to the following questions:

¹However, the unprepared reader may realize that school mathematics is a surprisingly subtle form of presenting much more involved structure—structure which becomes visible in the formalization.

²Some proof systems provide alternative proofs; some organize the proof in a modular way and the used lemmas are presented in separate subsections.
1. What is the home page of the system?
2. What are the books about the system?
3. What is the logic of the system?
4. What is the implementation architecture of the system?
5. What does working with the system look like?
6. What is special about the system compared to other systems?
7. What are other versions of the system?
8. Who are the people behind the system?
9. What are the main user communities of the system?
10. What large mathematical formalizations have been done in the system?
11. What representation of the formalization has been put in this paper?
12. What needs to be explained about this specific proof?

The editor states explicitly that “[o]ne of the main reasons for doing the comparison between provers is that I find it striking how different they can be” (p. 4). He provides an informative table (“consumer test”) marking with “+” and “−” which proof system provides the following features (some of them rather subjective):

- small proof kernel (‘proof objects’)
- calculations can be proved automatically
- extensible/programmable by the user
- powerful automation
- readable proof input files
- constructive logic supported
- logical framework
- typed
- decidable types
- dependent types
- based on higher order logic
- based on ZFC set theory
- large mathematical standard library
- statements about \( \mathbb{R} \)
- statements about \( \sqrt{ } \)

And, not even two of the 17 provers match in all these points.