Abstract. In this paper, we introduce a variety \( \mathbf{bdO} \) of Ockham algebras with balanced double pseudocomplementation, consisting of those algebras \((L; \wedge, \vee, f, *, +, 0, 1)\) of type \((2, 2, 1, 1, 0, 0)\) where \((L; \wedge, \vee, f, 0, 1)\) is an Ockham algebra, \((L; \wedge, \vee, *, +, 0, 1)\) is a double \(p\)-algebra, and the operations \(x \mapsto f(x), x \mapsto x^*\) and \(x \mapsto x^+\) are linked by the identities \([f(x)]^* = [f(x)]^+ = f^2(x), f(x^*) = x^{**}\) and \(f(x^+) = x^{++}\). We give a description of the congruences on the algebras, and show that there are precisely nine non-isomorphic subdirectly irreducible members in the class of the algebras via the Priestley duality. We also describe all axioms in the variety \(\mathbf{bdO}\), and provide a characterization of all subvarieties of \(\mathbf{bdO}\) determined by 12 nonequivalent axioms, identifying therein the biggest subvariety in which every principal congruence is complemented.

Keywords: double pseudocomplementation, Ockham algebra, Priestley duality, subdirectly irreducible.

1. Introduction

An Ockham algebra is a bounded distributive lattice \(L\) together with a dual endomorphism \(f : L \rightarrow L\). We shall denote by \(\mathbf{O}\) the class of Ockham algebras. Well known subclasses of Ockham algebras are Berman subclasses \(K_{p,q}\) in which the dual endomorphism \(f\) satisfies the condition \(f^{2p+q} = f^q\), for \(p \geq 1\) and \(q \geq 0\). For details of the basic properties of these algebras, we refer the reader to [1] or [4].

A distributive double \(p\)-algebra \((L; *, +, 0, 1)\) (or distributive lattice with double pseudocomplementation) is a distributive lattice \(L\) with a smallest element \(0\) and a biggest element \(1\) with a mapping \(* : L \rightarrow L\) such that \(x \wedge y = 0\) if and only if \(y \leq x^*\) and a mapping \(+ : L \rightarrow L\) such that \(x \vee y = 1\) if and only if \(y \geq x^+\). A fundamental congruence on a distributive double \(p\)-algebra is the relation \(G = G^* \wedge G^+\), in which \(G^*\) and \(G^+\) are defined by \((x, y) \in G^* \iff x^* = y^*\) and by \((x, y) \in G^+ \iff x^+ = y^+\). A special subclass of the class of distributive double \(p\)-algebras \((L; *, +)\) is the class of double Stone algebras in which the unary operations \(*\) and \(+\) meet the conditions: \(x^* \vee x^{**} = 1\) and \(x^+ \wedge x^{++} = 0\), for every \(x \in L\). For more details of properties of distributive double \(p\)-algebras, we refer the reader to [8].

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In our previous work [7], we introduced the notion of balanced pseudocomplemented Ockham algebras that is a subvariety which is contained in both the class of Ockham algebras and the class of pseudocomplemented algebras. Precisely, a balanced pseudocomplemented Ockham algebra \((L; f, *)\) is a bounded distributive lattice \(L\) endowed with two unary operations \(a \mapsto f(a)\) and \(a \mapsto a^*\) such that \((L; f)\) is an Ockham algebra, \((L; *)\) is a pseudocomplementated algebra, and the unary operations are linked by the identities: \(f(x^*) = x^{**}\) and \([f(x)]^* = f^2(x)\).

In our another paper [6], we investigated the class of Ockham algebras with double pseudocomplementation, namely those Ockham algebras that are endowed with the double pseudocomplement operations \(\ast\) and \(+\) such that the unary operations \(f, \ast, +\) are linked by the identities \([f(x)]^* = f(x^*)\) and \([f(x)]^+ = f(x^+)\).

Here we shall consider another particular class of algebras that is contained in both the class \(\mathbf{O}\) of Ockham algebras and the class \(\mathbf{dp}\) of double \(p\)-algebras. This subvariety is defined as follows.

**Definition.** An Ockham algebra with balanced double pseudocomplementation is an algebra \((L; f, *, +) \equiv (L; \wedge, \vee, f, \ast, +, 0, 1)\) of type \(\langle 2, 2, 1, 1, 0, 0 \rangle\) such that \((L; \wedge, \vee, f, 0, 1)\) is an Ockham algebra, \((L; \wedge, \vee, \ast, +, 0, 1)\) is a double pseudocomplemented algebra, and the unary operations \(f, \ast, +\) are linked by the following identities:

\[
\begin{align*}
(1) \ [f(x)]^* &= [f(x)]^+ = f^2(x); \\
(2) \ f(x^*) &= x^{**} \text{ and } f(x^+) = x^{++}.
\end{align*}
\]

We shall denote by \(\mathbf{bdO}\) the class of Ockham algebras with balanced double pseudocomplementation.

**Example 1.** Consider the algebra \((L; f, *, +)\) depicted as follows: