
Duality is a kind of loose and tricky notion relating mathematical (algebraic, or categorial) concepts. We sometimes know how to use it (as for instance in projective geometry, where the notions of “point” and “line” can be interchanged so that theorems about points in the plane lead to dual theorems about lines), but we hardly know how to explain it in abstract terms. Intuitively, two operations or concepts are dual if they can be interchanged, so that all results holding in one formulation also hold in the other, said to be the dual formulation. The problem is that duality has more than one sense, as shown more than a half century ago by W. H. Gottschalk in [7].

Dual tableaux, the topic of this book, refers to those proof structures treated in [11] as explicit variants of sequents, building on previous work by S. Kanger and E. W. Beth. Such tableaux can be seen as dual to familiar tableaux, but dual in which sense? A usual tableau proof of (or a proof of the validity for) a sentence $\phi$ is done by refuting the negation of $\phi$, that is, by showing that there exists a tableau for $\neg \phi$ such that each of its end points is contradictory. On the other hand, a dual-tableau proof of (or a proof of the validity for) a sentence $\phi$ is done by confirming $\phi$, that is, by showing that there exists a tableau for $\phi$ such that each of its end points is tautological. From this perspective, dual tableaux are a variant of Gentzen’s sequent systems (up to the upward or downward directions).

In [3], for instance, we find that for every finite (propositional or quantified) many-valued logic there is a precise duality between sequent calculi and tableau systems as investigated, for example, in [4]. Indeed, there is an exact correspondence between cut-free sequent calculi and closed tableaux in the realm of many-valued logics (for a constructive proof of this correspondence see [5]). Furthermore, such a duality is manifested in two different ways (cf. [3]).

If we consider a sentence as “satisfiable” in a given domain if there are assignments to its free variables that make it true, and as “valid” if every assignment to its free variables makes it true, it is clear that a sentence will be invalid if and only if its negation is satisfiable, and is unsatisfiable if and only if its negation is valid, but at the cost of requiring a classical-like notion of negation. Under such conditions, satisfiability and validity are dual notions, in one of the senses of duality explained in [7].

This bulky volume explores this notion of duality from a metamathematical viewpoint, and gathers together work of more than three decades, especially by the first author. The book is divided into seven parts. Part I, “Foundations”, contains three chapters. Chapter 1, “Dual Tableau for Classical First-Order Logic”, reviews basic material, including the Rasiowa-Sikorski proof system for classical first-order logic with identity, a brief discussion on duality, dual tableaux and Hilbert-style systems, the relationship between dual tableaux and Gentzen-style systems, and dual resolution. Chapter 2, “Dual Tableaux for Logics of Classical Algebras of
Binary Relations”, introduces the logics of binary relations and several related topics, emphasizing a method for proving soundness and completeness, as well as decision procedures, for some relational logics. Chapter 3, “Theories of Point Relations and Relational Model Checking”, studies relational logics, including model checking, verification and satisfaction in relational logics.


Part IV is dedicated to “Relational Reasoning in Logics of Information and Data Analysis”, a topic of more interest for computer science and engineering. Information logics are modal logics where certain information relations, such as similarity relations, backward inclusion relations, orthogonality relations, etc., govern the modal operators. Its five chapters cover dual tableaux for specific frames on such relations, for a modal fuzzy logic and for relative magnitude relations, to wit: Chapter 11, “Dual Tableaux for Information Logics of Plain Frames”, Chapter 12, “Dual Tableaux for Information Logics of Relative Frames”, Chapter 13, “Dual Tableau for Formal Concept Analysis”, Chapter 14, “Dual Tableau for a Fuzzy Logic” and Chapter 15, “Dual Tableaux for Logics of Order of Magnitude Reasoning”.

Part V, “Relational Reasoning about Time, Space, and Action”, discusses and defines systems of dual tableaux for logics of time and space and for propositional dynamic logics. The chapters in this part have the following titles: Chapter 16, “Dual Tableaux for Temporal Logics”, Chapter 17, “Dual Tableaux for Interval Temporal Logics”, Chapter 18, “Dual Tableaux for Spatial Reasoning” and Chapter 19, “Dual Tableaux for Logics of Programs”. While Chapters 17 and 19 essentially treat (multi) modal logics, Chapter 18 inherits from the tradition of S. Lesniewski’s mereology and several systems are discussed (not all of them typically modal-some are better expressed as first-order systems with specific classes of predicates).

Part VI, “Beyond Relational Theories”, puts together a miscellaneous collection of dual tableaux for threshold logics (Chapter 20, “Dual Tableaux for Threshold Logics”), for an infinite-valued valued logic (Chapter 21, ”Signed Dual Tableau for G"odel-Dummett Logic”), for a first-order many-valued logic (Chapter 22, “Dual Tableaux for First-Order Post Logics”), as well as a dual tableau for a non-Fregean logic, namely, for the propositional logic with identity (Chapter 23, ”Dual Tableau for Propositional Logic with Identity”) and for the propositional logic of binary decisions (Chapter 24, “Dual Tableaux for Logics of Conditional Decisions”).