The inclosure scheme and the solution to the paradoxes of self-reference

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Abstract All paradoxes of self-reference seem to share some structural features. Russell in 1908 and especially Priest nowadays have advanced structural descriptions that successfully identify necessary conditions for having a paradox of this kind. I examine in this paper Priest’s description of these paradoxes, the Inclosure Scheme (IS), and consider in what sense it may help us understand and solve the problems they pose. However, I also consider the limitations of this kind of structural descriptions and give arguments against Priest’s use of IS in favour of dialetheism. IS fails to identify sufficient conditions for having a paradox of self-reference. That means that, even if we identified a problem common to any reasoning satisfying IS, that problem would not explain why some of those reasonings are paradoxical and some others are not. Therefore IS cannot justify by itself the claim that some particular theory offers the best solution to the paradoxes of self-reference. We still need to consider aspects concerning the content and context of occurrence of every paradox.

Keywords Structure of the paradoxes of self-reference · Inclosure Scheme · Priest · Russell · Dialetheism

1 Introduction

In 1908, Bertrand Russell presented his theory of types as a tentative solution to a series of contradictions originated in different fields of knowledge. He considered
(1908, 59–61), among others, the liar paradox, Russell’s paradox and paradoxes due to Berry, König, Richard and Burali-Forti. Beyond the obvious differences displayed by all these paradoxes and the fact that they involved completely different concepts (truth, set, relation, definition, etc.), Russell (1908, 61) observed that they all shared some structural features. In a rather vague way, he described as “self-reference” or “reflexiveness” what all these paradoxes had in common. But he also provided more detailed descriptions.

The most relevant structural feature identified by Russell concerned the presence of some sort of vicious circularity. All these paradoxes involved some $a$ and some totality $\Omega$ (sometimes $a = \Omega$) such that $a$ was a possible member of $\Omega$, but one that could only be defined by appealing to all the members of $\Omega$. This clearly leads to a vicious circle in the definitions of $\Omega$ and $a$ since we need to know all the elements of $\Omega$ in order to define $a$, but we need to determine whether $a$ is an element of $\Omega$ in order to define all the elements of $\Omega$. In Russell’s paradox, for instance, $\Omega$ and $a$ are both “the class of all classes not belonging to themselves”: $\{y : y \notin dy\}$. In the liar paradox, $\Omega$ is the set $\{y : \text{true}(y)\}$ of all true sentences and $a$ is the sentence ‘$a \notin \Omega$’, which says of itself that it is not true or, in other words, that it is different from any member of $\Omega$. Something similar happens in the rest of paradoxes mentioned before, they all involve vicious circles concerning some totality, $\Omega$, and $a$, one of its possible members.

Given that vicious circles have always been suspicious in philosophy and $\Omega$ and $a$ are essential for us to draw a contradiction in the paradoxes of self-reference, we seem to have good reasons for rejecting entities like $\Omega$ and $a$. This was surely what prompted Russell to state his Principle of Vicious Circularity (PVC):

“Whatever involves all of a collection must not be one of the collection”; or, conversely: ‘If, provided a certain collection had a total, it would have members only definable in terms of that total, then the said collection has no total’” (Russell 1908, 63).

Any theory following this rule would exclude the joint existence of entities like $\Omega$ and $a$ and, a fortiori, any contradiction stemming from them. Russell’s Theory of Types embodied PVC and was mainly devised to avoid the formation of this kind of vicious circles. Had it been satisfactory, this theory would have achieved at least two goals. First of all, it would have provided a general solution to the paradoxes of self-reference, since PVC focuses on formal aspects present in all of them; and, secondly, it would have done so on the basis of relevant intuitions against vicious circles.

Summing up, we can articulate Russell’s strategy to solve these paradoxes in four steps: (1) A structural feature common to all paradoxes of self-reference is the presence of some totality, $\Omega$, and some element, $a$, whose respective definitions give rise to a vicious circle. (2) By ruling out entities like $\Omega$ and $a$, we can avoid the inference of contradictions in the paradoxes we are presently considering. (3) We have intuitive reasons for rejecting vicious circles and therefore for rejecting $\Omega$ and $a$. (4) A theory that bans entities like $\Omega$ and $a$ on the basis of PVC will provide a general solution to the paradoxes of self-reference grounded on intuitive reasons.

As far as I know Russell never claimed that vicious circles were fully responsible for the paradoxes of self-reference (presumably there are other relevant factors). Likewise, he did not claim that the circularity outlined before provided a complete structural description of this family of paradoxes. But, setting aside his own views, it seems clear that his foundational work on the subject raises several intriguing questions. For instance: Can we provide a complete structural description of the paradoxes