CONSTRUCTION OF FORM FACTORS OF COMPOSITE SYSTEMS BY A GENERALIZED WIGNER–ECKART THEOREM FOR THE POINCARÉ GROUP

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We generalize the previously developed relativistic approach for electroweak properties of two-particle composite systems to the case of nonzero spin. This approach is based on the instant form of relativistic Hamiltonian dynamics. We use a special mathematical technique to parameterize matrix elements of electroweak current operators in terms of form factors. The parameterization is a realization of the generalized Wigner–Eckart theorem for the Poincaré group, used when considering composite-system form factors as distributions corresponding to reduced matrix elements. The electroweak-current matrix element satisfies the relativistic covariance conditions and also automatically satisfies the conservation law in the case of an electromagnetic current.

Keywords: Wigner–Eckart theorem, Poincaré group, form factors, composite systems, relativistic Hamiltonian dynamics

1. Introduction

The development of correct quantitative methods for calculating composite-particle structure is an important line of investigations in particle physics. Modern experimental research on particle structure is performed for such energies and with such an accuracy that it calls for the relativistic theoretical methods for interpreting the results. An important feature of these methods is extracting kinematical parts and invariant parts (form factors) from the current matrix elements. A representation of the current matrix element in terms of form factors is called a parameterization. We note that precisely form factors (Lorentz invariant functions) can customarily be extracted from the experimental data.

In the relativistic theory of the description of composite systems, we can identify two main, but very different, approaches (see, e.g., [1] and the pictorial scheme in Fig. 11 in [2]). The first is the method of field theory. Based on the principles of quantum field theory, quantum chromodynamics (QCD), it is rightly regarded as the most consistent approach for solving this problem. But the standard perturbative QCD gives sufficiently reliable computational prescriptions only for the description of the so-called “hard” processes characterized by large momentum transfers and it does not permit calculating characteristics determined by “soft” processes. Moreover, there are strong indications that perturbative QCD fails when describing the current experimental facts in exclusive processes. In particular, this happens when describing elastic form factors of such well-studied composite systems as the pion, nucleon, and deuteron.

The second method in the relativistic theory of composite systems is gaining acceptance in recent years and is based on the direct realization of the algebra of the Poincaré group on the set of dynamical observables in the Hilbert state space of the system. This approach is called the theory of direct interaction

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or relativistic Hamiltonian dynamics (RHD). A discussion of this approach can be found, e.g., in [2], [3].
The method is based on starting positions quite different from those for the first method, which is one
reason why the results of these approaches are difficult to compare. We do not focus on the details of RHD
but refer the reader to [4] and the references therein.

Although the abovementioned methods for describing composite systems are quite different, it is nec-
essary to resolve three very important problems in both cases:

1. It is necessary to give the relativistic recipe for taking nonperturbative effects into account. We
note that in the RHD case (in the framework of which we operate), these effects are contained in
the very structure of the method (see [1], [4]).

2. It is necessary to construct the electroweak-current matrix element with correct transformation
properties and to perform the relativistic parameterization procedure for the current matrix ele-
ments, i.e., to extract the relativistic invariant form factors.

3. It is necessary to take into account that composite-system form factors, generally speaking, have
the sense of distributions (in the nonrelativistic case, too).

In simple cases, the parameterization can be easily obtained from general considerations. But in more
complicated cases (for example, for composite systems with an arbitrary total angular momentum), we
need special mathematical methods.

Our aim here is to construct a general parameterization method for the case of composite systems with
nonzero total spin values. The method was announced in [5]. Here, we give a consistent presentation of
our method, which allows applying it to specific composite systems. In some sense, our method is similar
to the method for canonically parameterizing one-particle matrix elements of field operators [6]. We note
that we are now dealing with RHD, not with field theory as in [6].

Conceptually, our parameterization method is a realization of the Wigner–Eckart theorem for the
Poincaré group, whereas form factors are the reduced matrix elements. This theorem is extensively used in
different fields of theoretical and mathematical physics, from the well-known theory of angular momentum
to the Hopf algebras and quantum superalgebras [7]. In the present paper, we use the generalized Wigner–
Eckart theorem for the Poincaré group, which is useful when composite-system form factors are considered
as distributions. The necessity of such a consideration was shown in [1], [4] in the study of form factors of
composite systems with zero total spin.

The paper is organized as follows. In Sec. 2, we describe the canonical parameterization procedure for lo-
cal operator matrix elements between one-particle states with arbitrary nonzero spin. This parameterization
extracts the reduced matrix element on the Poincaré group. We obtain formulas for the electromagnetic-
current matrix element for the spin-1/2 particle.

In Sec. 3, we show how to construct the electromagnetic current operator for a composite system of
two free particles with spin 1/2. The current matrix element is obtained in the basis where the center-of-
mass motion is separated. The electromagnetic properties of the system are described by reduced matrix
elements or the so-called free two-particle form factors having the sense of distributions. This means that
the static properties of the system, for example, are given by the weak limits as $Q^2 \to 0$ ($Q^2 = -q^2$ and
$q$ is the momentum transfer). Special attention is paid to the case of total spin one and zero total orbital
momentum. In this case, the electromagnetic properties are defined by four free two-particle form factors:
the charge, quadrupole, magnetic, and magnetic quadrupole form factors of the second kind (see [8] and
the references therein).

In Sec. 4, we develop the procedure for constructing the electromagnetic-current matrix element in the
case of two interacting particles. At each calculation step, the developed method for canonically parameter-
izing a current matrix element remains Lorentz covariant and preserves the current. The composite-system