GENERALIZED COHERENT STATES FOR OSCILLATORS ASSOCIATED WITH THE CHARLIER \( q \)-POLYNOMIALS

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We continue studying the generalized coherent states for oscillator-like systems associated with a given family of orthogonal polynomials. We consider the case of generalized oscillators generated by the Charlier \( q \)-polynomials.

Keywords: deformed oscillator, coherent state, orthogonal polynomials, Charlier \( q \)-polynomial

1. Introduction

It is difficult to overestimate the role played by coherent states in quantum theory. The basic concepts concerning the theory of coherent states can be found in [1], [2]. A vast bibliography including the period until 2000 is contained in [3]. In recent years, the interest in constructing and studying the coherent states of oscillator-like systems associated with families of orthogonal polynomials has increased (see, e.g., [4]–[7]).

In our previous works (see, e.g., [8], [9] and the references to our works therein), we proposed a new method for constructing oscillator-like systems associated with families of orthogonal polynomials, just as in the case of the usual boson oscillator associated with Hermite polynomials, and developed a scheme for constructing the generalized coherent states of such systems (see [9] for details). Here, we apply our construction to the Charlier \( q \)-polynomials.

2. Construction of a generalized oscillator

The generalized oscillator is defined by the commutation relations

\[
[a_\mu^-, a_\mu^+] = 2 \left( B(N+I) - B(N) \right), \quad [N, a_\mu^\pm] = \pm a_\mu^\pm
\]

(see the notation in [9]). We note that if \( B(N) \) is a polynomial in \( N \) (this holds in the case of the classical polynomials of both a continuous and a discrete argument), then the algebra \( A_\mu \) generated by the operators \( a_\mu^\pm \) and \( N \) with commutation relations (1) is called the algebra of the generalized oscillator associated with the canonical system of polynomials \( \{\psi_n(x)\}_{n=0}^{\infty} \) orthonormal with respect to the probability measure \( \mu \). In what follows, we also let the symbol \( A_\mu \) denote the oscillator itself. The center of \( A_\mu \) is generated by the element

\[
C = 2B(N) - a_\mu^- a_\mu^+.
\]

But if \( B(N) \) is not a polynomial but satisfies the recurrence relation

\[
\alpha B(N+I) + B(N) + \gamma B(N-I) = 0, \quad \alpha \neq 0, \quad \gamma \neq 0
\]

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The generalized oscillator algebra generated by these relations is denoted by the same symbol \( q \). The Charlier polynomials are orthogonal, which holds in the case of \( q \)-B polynomials of the second kind.

The recurrence relations for the polynomials \( \phi_n \) are given by

\[
\begin{align*}
\mu q^{-n} C_{n+1}^\mu(x | q) + (q^n - 1) C_{n-1}^\mu(x | q) &= (\mu + q)(q^n - 1 - x) C_n^\mu(x | q), \quad n > 0, \\
C_0^\mu(x | q) &= 1.
\end{align*}
\]

These polynomials are orthogonal,

\[
\sum_{s=0}^\infty C_n^\mu(q^{-s} | q) C_m^\mu(q^{-s} | q) \rho(s) = \frac{(q; q)_n}{\mu^n} a_{mn},
\]

with respect to the discrete positive measure

\[
\rho(s) = (\mu; q)_\infty \frac{\mu^s q^{s(s-1)}}{(q; \mu; q)_s}, \quad 0 < \mu, q < 1.
\]