TOPOLOGICAL EXCITATIONS IN A TWO-DIMENSIONAL SPIN SYSTEM WITH HIGH SPIN $s \geq 1$

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We construct a class of topological excitations of a mean field in a two-dimensional spin system represented by a quantum Heisenberg model with high powers of the exchange interaction. The quantum model is associated with a classical model (the continuous classical analogue) based on a Landau–Lifshitz-like equation, which describes large-scale fluctuations of the mean field. On the other hand, the classical model in the case of spin $s$ is a Hamiltonian system on a coadjoint orbit of the unitary group $SU(2s+1)$.

We construct a class of mean-field configurations that can be interpreted as topological excitations because they have fixed topological charges. Such excitations change their shapes and grow, conserving energy.

Keywords: order parameter, mean field, effective Hamiltonian, coadjoint orbit

1. Introduction

According to [1], there is no ferromagnetic or antiferromagnetic order in the one- and two-dimensional isotropic Heisenberg models with finite-range interactions at a nonzero temperature. This statement follows from Bogoliubov’s inequality in the general case. Here, we construct excitations that cause a destruction of a long-range nematic or mixed ferromagnetic–nematic order. This extends the results in [2].

We model a planar magnet by a square atomic lattice with the same spin $s$ at each site. We describe this two-dimensional spin system by a generalized Heisenberg Hamiltonian, taking high powers of the exchange interaction ($\hat{S}_n, \hat{S}_{n+\delta}$) into account, where $\hat{S}_n$ is a vector of spin operators at the site $n$. By a mean-field approximation, we obtain a classical long-range equation from the quantum Heisenberg equation.

An equation for a mean field (the field of magnetization and multipole moments) is a Hamiltonian equation on a coadjoint orbit of the Lie group. At the same time, it is a generalization of the well-known Landau–Lifshitz equation for a magnetization field. In this context, we obtain effective Hamiltonians for the magnetic system in question. Using the Kähler structure of coadjoint orbits, we construct effective Hamiltonians such that their minimums are proportional to topological charges of excitations. In addition, we produce the mean-field configurations that minimize the Hamiltonians.

2. Quantum and classical models

We consider a spin system in the form of a planar atomic$^1$ lattice with the same spin $s$ at all sites. We assign three spin operators ($\hat{S}_n^1, \hat{S}_n^2, \hat{S}_n^3$) = $\hat{S}_n$ to each atom $n$; these operators satisfy the standard commutation relations

$$[\hat{S}_n^a, \hat{S}_m^b] = i\varepsilon_{abc}\hat{S}_n^c\delta_{nm},$$

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$^1$Here and hereafter, by atom, we mean not only an atom in the exact sense but also a molecule, an ion, or any compound of atoms regarded as an entity with the spin $s$.

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where \(a, b, c\) range the values \(\{1, 2, 3\}\) and \(\delta_{nm}\) is the Kronecker symbol.

### 2.1. Generalized Heisenberg Hamiltonians.

We consider the case of high spins \(s \geq 1\). In this case, we can describe the system by the bilinear-biquadratic Hamiltonian

\[
\hat{H}^2 = -\sum_{n, \delta}(J(\mathbf{S}_n \cdot \mathbf{S}_{n+\delta}) + K(\mathbf{S}_n \cdot \mathbf{S}_{n+\delta})^2).
\]

Here, \(\delta\) ranges the nearest-neighbor sites, \(n\) ranges all lattice sites, and the constants \(J\) and \(K\) denote exchange integrals. In the case of spin \(s \geq 3/2\), this Hamiltonian can also include the bicubic exchange, namely,

\[
\hat{H}^3 = -\sum_{n, \delta}(J(\mathbf{S}_n \cdot \mathbf{S}_{n+\delta}) + K(\mathbf{S}_n \cdot \mathbf{S}_{n+\delta})^2 + L(\mathbf{S}_n \cdot \mathbf{S}_{n+\delta})^3),
\]

where \(L\) denotes the corresponding exchange integral.

We write a generalized Heisenberg Hamiltonian for the system of an arbitrary spin \(s\) or a spin greater than \(s\). This Hamiltonian contains all powers of the exchange interaction up to \(2s\). It can be reduced to a bilinear form if we take the \((2s+1)\)-dimensional space of irreducible representation of the group \(SU(2)\). The spin operators \(\{\hat{S}_n^a\}\) over this space generate a complete associative matrix algebra, which has a sufficient number of operators to reduce the corresponding Hamiltonian to a bilinear form.

For example, in the case of spin \(s = 1\), the appropriate representation space is three-dimensional, and we choose a canonical basis in the form \(\{|+1\rangle, |-1\rangle, |0\rangle\}\). The spin operators \(\{\hat{S}_n^a\}\) generate the algebra \(\text{Mat}_{3 \times 3}\). To form a basis in the algebra, we take the tensor operators of weight 2

\[
\hat{Q}_{n}^{ab} = \hat{S}_{n}^{a}\hat{S}_{n}^{b} + \hat{S}_{n}^{b}\hat{S}_{n}^{a}, \quad a \neq b,
\]

\[
\hat{Q}_{n}^{22} = (\hat{S}_{n}^{1})^2 - (\hat{S}_{n}^{2})^2, \quad \hat{Q}_{n}^{20} = \sqrt{3}\left((\hat{S}_{n}^{3})^2 - \frac{2}{3}\right)
\]

in addition to the spin operators. The introduced operators are called quadrupole operators.

In the case of spin \(s = 3/2\), the appropriate representation space is four-dimensional, and \(\{|+3/2\rangle, |+1/2\rangle, |-1/2\rangle, |-3/2\rangle\}\) is a canonical basis. We complete the associative matrix algebra \(\text{Mat}_{4 \times 4}\) of \(\{\hat{S}_n^a\}\) by the tensor operators of weights 2 and 3, defining them by the formulas

\[
\hat{T}_{n}^{ab} = \frac{\sqrt{5}}{2\sqrt{3}}(\hat{S}_{n}^{a}\hat{S}_{n}^{b} + \hat{S}_{n}^{b}\hat{S}_{n}^{a}), \quad a \neq b,
\]

\[
\hat{T}_{n}^{22} = \frac{\sqrt{5}}{2\sqrt{3}}((\hat{S}_{n}^{1})^2 - (\hat{S}_{n}^{2})^2), \quad \hat{T}_{n}^{20} = \frac{\sqrt{5}}{2}\left((\hat{S}_{n}^{3})^2 - \frac{5}{4}\right),
\]

\[
\hat{T}_{n}^{a3} = (\hat{Q}_{n}^{a2}\hat{S}_{n}^{3} + \hat{S}_{n}^{3}\hat{Q}_{n}^{a2}), \quad a, b \in \{1, 2\}, \quad a \neq b,
\]

\[
\hat{T}_{n}^{ab} = \frac{1}{\sqrt{6}}((\hat{S}_{n}^{a})^2\hat{S}_{n}^{b} + \hat{S}_{n}^{b}(\hat{S}_{n}^{a})^2 + \hat{S}_{n}^{a}\hat{S}_{n}^{b}\hat{S}_{n}^{a} - (\hat{S}_{n}^{b})^3),
\]

\[
\hat{T}_{n}^{33} = \frac{1}{\sqrt{10}}(\hat{Q}_{n}^{33}\hat{S}_{n}^{3} + \hat{S}_{n}^{3}\hat{Q}_{n}^{33} + \sqrt{3}(\hat{Q}_{n}^{20}\hat{S}_{n}^{a} + \hat{S}_{n}^{a}\hat{Q}_{n}^{20})),
\]

\[
\hat{T}_{n}^{30} = \frac{1}{12}(41\hat{S}_{n}^{3} - 20(\hat{S}_{n}^{3})^3).
\]

We call the tensor operators of weight 3 sextupole operators. In what follows, we let \(\{\hat{P}_{n}^a\}\) denote all tensor operators over the chosen representation space.