WHICH SCORING RULE MAXIMIZES CONDORCET EFFICIENCY UNDER IAC?

ABSTRACT. Consider an election in which each of the \( n \) voters casts a vote consisting of a strict preference ranking of the three candidates \( A \), \( B \), and \( C \). In the limit as \( n \to \infty \), which scoring rule maximizes, under the assumption of Impartial Anonymous Culture (uniform probability distribution over profiles), the probability that the Condorcet candidate wins the election, given that a Condorcet candidate exists? We produce an analytic solution, which is not the Borda Count. Our result agrees with recent numerical results from two independent studies, and contradicts a published result of Van Newenhizen (Economic Theory 2, 69–83. (1992)).

KEY WORDS: Condorcet efficiency, scoring systems, Borda count, impartial anonymous culture, voting.

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1. INTRODUCTION

We wish to consider elections in which \( n \) voters select a winner from among the three candidates \( A \), \( B \), and \( C \). We assume that each voter expresses, as his or her vote, a strict, complete, and transitive preference ranking of the candidates; in particular, no voter expresses indifference between any two candidates. Each voter chooses, then, from among six possible rankings:

\[
\begin{array}{cccccc}
A & C & C & B & B & A \\
B & B & A & A & C & C \\
C & A & B & C & A & B \\
\end{array}
\]
Here, each $n_i$ is equal to the number of voters who express the associated ranking. Thus, any 6-tuple $P = (n_1, n_2, n_3, n_4, n_5, n_6)$ of non-negative integers that sum to $n$ tells us how many voters chose each of the rankings in a given election. Such a tuple is known as a *profile*.

Numerous criteria have been suggested to help select which voting systems, among various systems for choosing winners from profiles, best reflect the cumulative will of the electorate. One of the most common of these is the *Condorcet Criterion*. If $U$ and $V$ are candidates, let $U >^C V$ indicate that candidate $U$ defeats $V$ in the pairwise majority election between these two (strictly more voters ranked $U$ over $V$ than ranked $V$ over $U$); we’ll use $U \geq^C V$ for the corresponding weak relation. A candidate is the *Condorcet winner* if she defeats each other candidate in pairwise majority elections. It is well known (Condorcet, 1989; Sommerlad and McLean, 1989) that a Condorcet winner need not exist. However, many feel that a reasonable voting system should elect the Condorcet winner whenever such a candidate exists; this is the *Condorcet Winner Criterion*.

The cost of completely meeting this criterion can be high, however, because it forces us to sacrifice certain other desirable requirements (Saari, 1995). So it has become common to consider, as a measure of partial fulfillment, the *Condorcet efficiency* of a voting system $S$, which is the conditional probability that $S$ elects the Condorcet winner, given that a Condorcet winner exists.

Condorcet efficiency depends, of course, on the underlying probability distribution describing the likelihood that various profiles are observed. Many such distributions have been considered, but the two most common assumptions are:

*Impartial Culture (IC)*: voters choose a preference ranking randomly and independently with probability $1/6$ of choosing any particular ranking.

*Impartial Anonymous Culture (IAC)*: each possible preference profile is equally likely.$^1$