ABSTRACT. We examine the maximal-element rationalizability of choice functions with arbitrary domains. While rationality formulated in terms of the choice of greatest elements according to a rationalizing relation has been analyzed relatively thoroughly in the earlier literature, this is not the case for maximal-element rationalizability, except when it coincides with greatest-element rationalizability because of properties imposed on the rationalizing relation. We develop necessary and sufficient conditions for maximal-element rationalizability by itself, and for maximal-element rationalizability in conjunction with additional properties of a rationalizing relation such as reflexivity, completeness, \( P \)-acyclicity, quasi-transitivity, consistency and transitivity.

KEY WORDS. choice functions, maximal-element rationalizability

JEL CODE: D11.

1. INTRODUCTION

The notion of rational choice as optimizing choice dates at least as far back as Robbins (1932, 1935, p. 93), who asserted that “there is a sense in which the word rationality can be used which renders it legitimate to argue that at least some rationality is assumed before human behaviour has an economic aspect—the sense, namely, in which it is equivalent to ‘purposive’. . . .” The elaborate edifice of revealed preference

* Financial support through grants from the Social Sciences and Humanities Research Council of Canada, the Fonds pour la Formation de Chercheurs et l’Aide à la Recherche of Québec, and a Grant-in-Aid for Scientific Research for Priority Areas from the Ministry of Education, Culture, Sports, Science and Technology of Japan is gratefully acknowledged. Thanks are also due to the editor and the two referees for the opportunity to improve the exposition of this paper.
theory à la Samuelson (1938, 1947, Chapter V, 1948, 1950) and Houthakker (1950) was the first formal treatment of this notion of rational choice. The strong axiom of revealed preference due to Houthakker was meant to be a sufficient condition for the demand function of a competitive consumer to be derived by means of the optimization of an underlying preference ordering or utility function. This line of research has been further explored by Arrow (1959), Richter (1966, 1971), Hansson (1968), Sen (1971), Suzumura (1976, 1977, 1983, Chapter 2), Bossert, Sprumont and Suzumura (2004, 2005) and many others. Note, however, that the optimization of a single underlying preference ordering or utility function is not the only way of giving substance to the Robbinsian notion of ‘purposive behaviour.’ An alternative model of purposive behaviour may require that there exist multiple preference orderings such that an alternative chosen from an option set is obtained by means of the maximization of the intersection of these underlying preference orderings. If these orderings may be construed as the individual preference orderings, the set of chosen options is nothing other than the set of Pareto-efficient options. Alternatively, the underlying preference orderings may be construed as potential preference orderings which a decision-maker may have in the future. In this case, the set of chosen options consists solely of those options which will never be rejected whichever potential preference ordering may materialize in the future. These examples will suffice to illustrate that the exploration of the Robbinsian notion of rational choice in terms of the maximal elements according to an underlying preference relation (which is not necessarily complete) is a worthwhile and important subject to be explored. See Schwartz (1976) and Sen (1997) for further motivation of exploring maximal-element rationalizability rather than greatest-element rationalizability. This paper is devoted to this issue. The analysis of necessary and sufficient conditions for maximal-element rationalizability by general relations and on arbitrary domains has, so far, not been explored thoroughly.

There are three identifiable domains of a choice function of historical importance. The first of these presupposes that the