DIFFUSION APPROXIMATION WITH EQUILIBRIUM FOR EVOLUTIONARY SYSTEMS SWITCHED BY SEMI-MARKOV PROCESSES

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We consider an evolutionary system switched by a semi-Markov process. For this system, we obtain an inhomogeneous diffusion approximation results where the initial process is compensated by the averaging function in the average approximation scheme.

Introduction

Dynamical systems described by an evolution equation is a classical topic in stochastic modeling. Asymptotic analysis of such systems was studied by several authors (see, e.g., [1–5]).

The usual asymptotic approach, in the diffusion approximation scheme, consists of normalizing the process about an equilibrium point obtained by a balance condition with respect to the equilibrium distribution. Another diffusion approximation can be obtained by considering a fluctuation with respect to the average process. In a previous work, we studied evolutionary systems with Markov switching in two cases [6] (the case where the average process is a deterministic function and the case where the average is a stochastic process).

In the present paper, we compensate the initial process by an averaging deterministic function instead of an equilibrium point (see, e.g., [6]) and obtain an inhomogeneous diffusion approximation result.

In Sec. 2, we describe processes used in our analysis. In Sec. 3, we present the result (Theorem 1), and in Sec. 4 we present the proof of this theorem.

1. Preliminaries

Let \( E \) be a Polish space and let \( \mathcal{C} \) be its Borel \( \sigma \)-algebra. We call the measurable space \( (E, \mathcal{C}) \) a standard state space.

The semi-Markov continuous stochastic system is considered in the series scheme with small series parameter \( \varepsilon > 0, \varepsilon \to 0 \), described by a solution of the evolution equation in \( \mathbb{R}^d \)

\[
\frac{d}{dt} U^\varepsilon(t) = a_{\varepsilon}\left(U^\varepsilon(t); x\left(\frac{t}{\varepsilon^2}\right)\right),
\]

where \( u \in \mathbb{R}^d \) and \( x \in E \).

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The semi-Markov switching process $x(t), t \geq 0$, on the standard state space $(E, \mathcal{E})$ is given by the semi-Markov kernel

$$Q(x, B, t) = P(x, B)F_x(t), \quad (3)$$

where $x \in E$, $B \in \mathcal{E}$, and $t \geq 0$, and is supposed to be uniformly ergodic with stationary distribution $\pi(B)$, $B \in \mathcal{E}$, satisfying the relation

$$\pi(dx) = \rho(dx)\frac{m(x)}{m}, \quad (4)$$

where $\rho(B)$, $B \in \mathcal{E}$, is the stationary distribution of the imbedded Markov chain $x_n$, $n \geq 0$, given by the stochastic kernel

$$P(x, B) := \mathbb{P}(x_{n+1} \in B \mid x_n = x). \quad (5)$$

In addition,

$$m(x) := \int_0^{\infty} \overline{F}_x(t)dt, \quad \overline{F}_x(t) := 1 - F_x(t), \quad m := \int_E \rho(dx)m(x). \quad (6)$$

It is well known (see, e.g., [3]) that, under some additional conditions, the stochastic system $U^\varepsilon(t), t \geq 0$, converges weakly to the deterministic average process $\hat{U}(t), t \geq 0$, defined by a solution of the average evolution equation

$$\frac{d}{dt} \hat{U}(t) = \hat{a}(\hat{U}(t)), \quad (7)$$

with the average velocity

$$\hat{a}(u) := \int_E \pi(dx)a(u, x). \quad (8)$$

It is natural that the fluctuation of the stochastic system around the average process can be described by the diffusion process (see [6]). The diffusion approximation scheme for the semi-Markov continuous stochastic system (1) is considered here for the centered normalized process

$$\xi^\varepsilon(t) = \varepsilon^{-1}[U^\varepsilon(t) - \hat{U}(t)]. \quad (9)$$

3. Main Results

The main result is formulated as follows:

**Theorem 1.** Let the stochastic evolutionary system (9) be defined by relations (1)–(9) and let the following conditions be satisfied: