BRIEF COMMUNICATIONS

CONDITIONS FOR BALANCE BETWEEN SURVIVAL AND RUIN

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Let $\xi_t$ be a classic risk process or a risk process with stochastic premiums. We establish conditions for balance between ruin and survival in the case of zero initial capital $u = 0$ (ruin probability $q = \Psi(0) = 1/2$, survival probability $p = 1 - q = 1/2$) and determine premium estimates under these conditions.

In [1, p. 47], principles of premium calculation were considered for the risk process $\xi_t = P_t - S_t$, where $S_t$ is the claim process and $P_t$ is the compensatory premium function on $[0; t]$. If $t = 1$, then, for $S = S_1$ and $P = P_1$, using the distribution function of claims $S$ on $[0; 1]$, namely,

$$G_S(x) = P\{S < x\}, \quad ES = \int xdG_S(x), \quad \sigma^2(S) = DS < \infty,$$

one introduces functionals for the estimation of the compensatory premium $P_1 = F_1[G_S(x)]$.

We restrict ourselves to the first three principles, which are related to the moments of claims $ES$ and $\sigma^2(S)$:

(i) the expected-value principle;

(ii) the standard-deviation principle;

(iii) the variance principle.

According to these principles, the compensatory premium estimators are given by the relations

$$P_{(a)} = ES + \alpha_0 ES = (1 + \alpha_0)ES,$$

$$P_{(b)} = ES + \alpha \sigma(S), \quad \sigma(S) = \sqrt{DS}, \quad \sigma = \frac{1}{\sqrt{DS}},$$

$$P_{(c)} = ES + \beta \sigma^2(S),$$

where $\alpha_0$, $\alpha$, and $\beta$ are positive level parameters of the estimators.

First, we determine the compensatory estimators for $C$ under the balance condition $p = q = 1/2$. These estimators are marked by the index $\ast$ because, in a certain case, they serve as a starting point in estimation under the condition $p \neq q$.

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1. Consider the classic reserve risk process \( \xi_t = C t - S_t \), where \( C > 0 \),

\[
S_t = \sum_{k \leq N_t} Y_k
\]
is the claim process, \( N_t \) is a Poisson process with intensity \( \lambda > 0 \), and

\[
E\!S = \sum_{r \geq 1} \frac{1}{r!} e^{-r} \lambda^r E\!\left[ \sum_{k=1}^{r} Y_k \right] = \lambda E\!Y_1, \quad E\!Y_1 = \mu_1 < \infty.
\]

(2)

\[
DS = \lambda D Y_1 = \lambda [\mu_2 - \mu_1^2], \quad \mu_2 = E\!Y_1^2 < \infty.
\]

Let us clarify the relationship between the ruin probability \( q = \Psi(0) \) and the survival probability \( p = 1 - q \) for \( u = 0 \) with the following risk premium under the condition \( E\!\xi_1 = C - \lambda \mu_1 > 0 \):

\[
\delta = \frac{C - E\!S}{E\!S} = \frac{C - \lambda \mu_1}{\lambda \mu_1} > 0, \quad E\!S = \lambda \mu_1 > 0.
\]

(3)

Denote the value of risk premium by \( \delta_* \) for \( p_+ = q_+ \), by \( \delta_0 > \delta_* \) for \( p_+ > q_+ \), and by \( \delta_0 < \delta_* \) for \( p_+ < q_+ \).

On the basis of the established relationship between \( p_+, q_+ \), and \( \delta \), we can also estimate the premium value for \( C \).

As in [2], for convenience, we introduce a claim surplus process instead of the reserve risk process \( \xi_{t,u} = u + \xi_t \) as follows:

\[
\xi_t = S_t - C t, \quad \xi_0 = 0, \quad C > 0, \quad m = E\!\xi_1 = \lambda \mu_1 - C < 0.
\]

(4)

Under condition (3) (i.e., for \( m < 0 \)), the absolute maximum

\[
\xi^+ = \sup_{0 \leq t < \infty} \xi_t
\]

has a nondegenerate distribution, which defines the ruin probability

\[
\Psi(u) = P\{\xi^+ > u\}, \quad u \geq 0, \quad \Psi(0) = q_+, \quad p_+ = 1 - q_+.
\]

(5)

The relationship between \( \delta \), \( p_+ \), and \( q_+ \) is established by the following theorem (see [2, p. 372]):

**Theorem 1.** Under condition (3), the following relation holds for process (4):

\[
\delta = \frac{C - \lambda \mu_1}{\lambda \mu_1} = \frac{p_+}{q_+}, \quad p_+ + q_+ = 1.
\]

(6)

Under the condition of balance between survival and ruin (\( p_+ = q_+ \), an analog of “fair play”), it follows from (6) that an estimate for the intensity of premium arrivals is determined by the value of \( C_* \) in the relation

\[
\delta = \delta_* = \frac{C - E\!S}{E\!S} = 1 \Rightarrow C = C_* = 2E\!S = 2\lambda \mu_1, \quad P^*(t) = C_* t.
\]

(7)