AN ITEM RESPONSE MODEL FOR NOMINAL DATA BASED ON THE RISING SELECTION RATIOS CRITERION

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Complete response vectors of all answer options in multiple-choice items can be used to estimate ability. The rising selection ratios criterion is necessary for scoring individuals because it implies that estimated ability always increases when the correct alternative is selected. This paper introduces the generalized DLT model, which assumes rising selection ratios and uses three parameters to describe each incorrect alternative. It is shown that the nominal categories and the Thissen and Steinberg model do not meet the criterion. A simulation study on goodness of recovery and an example with real data are also included.

Key words: multiple-choice items, nominal data, rising selection ratio, DLT, nominal categories model, Thissen & Steinberg model.

Complete response vectors of all answer options in multiple-choice items may provide more information about the estimated ability than dichotomous (right/wrong) responses (Abrahamowicz and Ramsay, 1992). Complete, or polytomous, responses include the correct option and the distractors. However, to estimate ability from polytomous responses it is important that the data satisfy the rising selection ratios criterion (RSRC), which was proposed by Love (1997) as a standard for evaluating data from multiple-choice items in a nonparametric framework.

The purpose of this paper is to introduce a new item response model for nominal data motivated by the RSRC. The model is labeled the generalized distractor rejection model based on a latent trait (DLT), because it assumes that the individuals respond by rejecting the incorrect alternatives (or distractors). Moreover, a latent continuous variable is used to classify the individuals. The original DLT model (Revuelta, 2004) used one parameter to describe each distractor whereas the generalized model uses three.

The most widely used polytomous models for multiple-choice items are the nominal categories model (NCM; Bock, 1972) and the Thissen and Steinberg model (TSM; Thissen and Steinberg, 1984; Thissen, Steinberg, and Fitzpatrick, 1989). These models satisfy the RSRC only if some additional constraints are imposed on the parameters. This paper describes such a constraint for the NCM and shows that DLT is a generalization of the constrained nominal categories model.

The rest of the paper is organized as follows: Section 1 presents the RSRC, its implications for estimating ability and its relation to previous developments in nonparametric item-response theory. Section 2 introduces DLT on the basis of the RSRC. In Section 3, the three-parameter logistic model (3pl; Birnbaum, 1968), the NCM and the TSM are evaluated against this criterion, and it is shown that they are closely related to DLT. Section 4 is devoted to the estimation of subject and item parameters. Section 5 presents a simulation on goodness of recovery. Section 6 describes an application to real data that shows how DLT applies in practice. Finally, Section 7 concludes the paper and suggests some lines of future research.
1. Rising Selection Ratios

A multiple-choice item is composed of A response alternatives. In the following, alternative 1 is correct and the others are distractors. Subindex $y = 1, \ldots, A$ is used to denote the alternatives, and $r = 2, \ldots, A$ refers to distractors.

1.1. Definition

Let $\theta$ be the ability parameter. The selection ratio for distractor $r$ is given by:

$$\psi_r(\theta) = \frac{\text{Probability of selecting 1}}{\text{Probability of selecting } r}.$$ 

Love (1997) proposed that the items must satisfy the RSRC:

$$\psi_r(\theta)$$ is monotone nondecreasing in $\theta$ for any $r$,

and suggested the $\gamma$ correlation coefficient (Goodman and Kruskal, 1979) to evaluate this criterion.

1.2. Relations of RSRC to Monotonic Homogeneity

The RSRC is an application to multiple-choice items of the monotonic homogeneity requirement, widely studied in nonparametric item-response theory (Mokken, 1997). Monotonic homogeneity holds for dichotomous data if the probability of a correct response is monotonically nondecreasing in $\theta$. Monotonic homogeneity has been extended to cover the case of more than two ordered response categories (Molenaar, 1997). Responses are ordered if the response in a higher category is indicative of a higher ability. Monotonic homogeneity holds for ordered responses if the probability of giving response $y$ or above is monotonically nondecreasing in $\theta$ for any $y$.

This paper focuses on the analysis of more than two nominal response categories, when monotonic homogeneity for ordered responses is inappropriate. In this kind of data, the selection of the correct response is indicative of a higher ability than the selection of any of the distractors. However, distractors are not ordered. Suppose that the item is decomposed into $A - 1$ subitems. Subitem $r$ is formed by the correct alternative and distractor $r$, and the response is therefore restricted to the set $\{1, r\}$. By definition, the RSRC is equivalent to monotonic homogeneity for all the subitems. Lemma 1 shows that the RSRC is a stronger condition than monotonic homogeneity for the complete item.

Lemma 1. (Monotonic homogeneity and the RSRC) . The RSRC implies monotonic homogeneity. The reverse is not true.

Demonstration: Let $P_1$ and $P_r$ be the probabilities of the correct option and distractor $r$. The RSRC implies that, for any $r$, the derivative of the selection ratio is nonnegative:

$$\psi_r'(\theta) = \frac{P_1' P_r - P_1 P_r'}{P_r^2} \geq 0.$$ 

In consequence $P_1' P_r \geq P_1 P_r'$. Summing over all the alternatives: $P_1' \geq P_1 \sum_r P_r'$, which can be written: $[\log P_1]' \geq \sum_r P_r' [\log P_r]' = 0$. Thus, $P_1' \geq 0$ and monotonic homogeneity holds. On the other hand, $P_1' P_r \geq P_1 P_r'$ can be written as $[\log P_1]' \geq [\log P_r]'$. If $0 < [\log P_1]' < [\log P_r]'$ for some $r$ monotonic homogeneity holds but the RSRC does not.