OPTIMAL PRICING, PRODUCTION, AND SALES POLICIES FOR NEW PRODUCT UNDER SUPPLY CONSTRAINT*

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Abstract This paper considers a firm that sells a durable product with a given market potential. The purpose of the firm is to maximize its profit by determining how much capacity to install before the sales horizon, how many products to produce in accordance with the capacity, and how many products to sell by pricing. Appealing to Pontryagin maximum principle in control theory, the authors obtain the closed-forms of all optimal decisions the firm should make. Furthermore, the optimal production rate and optimal sales rate are both equal to the demand rate, which is caused by the optimal pricing policy during the whole horizon, and the optimal pricing path is increasing with the cost of installing a unit of capacity. Furthermore, numerical analysis reveals the visual impression of the relationship of the parameters.

Keywords Capacity constraint, inventory, new product, pricing.

1 Introduction
When a firm introduces a new product, it must trade off the cost of capacity and inventories with the revenues from the product’s sale over its life cycle. There are some important oper-
ations decisions the firm must determine, such as how much capacity it should install, how it produces the products and how it sells the product by choosing an appropriate pricing policy. First of all, the firm should determine the production capacity by specifying an exogenously defined demand trajectory for the new product over time. Although the chosen level of capacity does not affect the demand dynamics directly, it can affect the firm on determining the pricing policy, production policy, and sales policy. Hence, choosing an appropriate capacity is a very important decision. After determining the capacity, the firm then determines how to produce and how to sell the product to maximize its profit.

Inventory and production models have been studied for a long history. Pekelman[1] treated the problem of simultaneously determining the optimal price and production rate of a firm over a known horizon. He characterized the optimal intertemporal price and production decisions depending on the sum of the adjoint variable of the inventory level and the Lagrange multiplier of the state constraint. Feichtinger and Hartl[2] analyzed an intertemporal optimization model of optimal pricing and production. They obtained the characterizations of the optimal policy appealing to Pontryagin maximum principle[3]. In their model, the nonnegative constraint for inventory was relaxed by allowing for shortages that were penalized by shortage costs. Thompson, et al.[4] also analyzed an inventory model with simultaneous price and production decisions. They obtained the strong planning and strong forecast horizon which could be used to decompose the original problem into several smaller problems to solve. Cheryl[5] was concerned with a profit maximizing firm that derived the optimal price for his level of output, level of inventory and composition of productive capacity of over time. In the above literature, demand functions are all assumed to be just time and price-dependent. However, in many situations, the demand function is also dependent on the cumulative sale. In our model, demand function is an innovation diffusion which is related to cumulative demand and cumulative sale.

In the last several decades, the innovation diffusion has been studied by many researchers. The much younger field of quantitative modelling of innovation diffusion in marketing started with the introduction of the simple diffusion model by Bass[6]. Next, we briefly review Bass model about the diffusion demand by time \( t \). Let the conditional likelihood of adoption be increasing linearly in the number of existing adopters, i.e., \( \alpha + \beta D(t) \), where \( \alpha \) and \( \beta \) are parameters and \( D(t) \) denotes cumulative demand up to time \( t \). The demand rate, \( d(t) \), is

\[
d(t) = \left[ N - D(t) \right] \left[ \alpha + \beta D(t) \right],
\]

where \( D(t) = \int_0^t d(s) \, ds \) and \( N \) is the fixed market potential. Note that this model implicitly assumes a homogeneous population, because all individuals in the population are equally “susceptible”.

Robinson and Lakhani[7] introduced price into Bass model in a multiplicative way as follows:

\[
d(t) = \left[ N - D(t) \right] \left[ \alpha + \beta D(t) \right] e^{-\rho p(t)},
\]

where \( p(t) \) is price, and \( \rho \) is a price sensitivity parameter. (Hereafter we will eliminate the time argument whenever there is no confusion). This model was subsequently used by Dolan and