Vibration and power flow analysis of periodically reinforced plates

H. A. XU & W. L. LI*

Department of Mechanical Engineering, Wayne State University, 5050 Anthony Wayne Drive, Detroit, MI 48202, U.S.A.

Received January 4, 2011; accepted February 17, 2011; published online March 31, 2011

In this paper an analytical method is proposed to investigate the vibration and power flows of periodically reinforced plate with general boundary conditions. Both the plate and stiffening beams are modeled as 3D structural components, and the couplings at the interfaces are specified in terms of 3D elastic joints. The displacement function for each stiffening beam is expressed as a modified Fourier cosine series, and the transverse and in-plane displacements for the plate are similarly expressed as the 2D versions of the modified Fourier cosine series expansions. The unknown Fourier coefficients are calculated using the Rayleigh-Ritz technique. The key advantages of the proposed method include: 1) it is capable of dealing with arbitrary boundary and coupling conditions, 2) it allows modeling any number of reinforcing beams with arbitrary lengths, and 3) the structural intensity, power flows, and kinetic energy distributions are readily calculated analytically from the displacement functions through appropriate mathematical (differential) operations, to name a few. The power flow characteristics of periodically reinforced plates are studied against various influencing factors, such as, plate and beam boundary conditions, coupling conditions, excitation locations, and dislocations resulting from minor misplacement of a reinforcing beam.

vibrations, reinforced plates, periodic structures, analytical methods


Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>vector of Fourier coefficients</td>
</tr>
<tr>
<td>$A_{nm}$, $B_{nm}, C_{nm}$</td>
<td>Fourier expansion coefficients of plate</td>
</tr>
<tr>
<td>$A_{b,n}$, $B_{b,n}$</td>
<td>Fourier coefficients of transverse displacement of beam</td>
</tr>
<tr>
<td>$A_{a,n}$, $B_{a,n}$</td>
<td>Fourier coefficients of axial displacement of beam</td>
</tr>
<tr>
<td>$A_{t,n}$, $B_{t,n}$</td>
<td>Fourier coefficients of torsional displacement of beam</td>
</tr>
<tr>
<td>$a$</td>
<td>length of plate</td>
</tr>
<tr>
<td>$l_{ma}, l_{nb}, l_{mc}, l_{nd}, l_{me}, l_{nf}$</td>
<td>Fourier expansion coefficients of plate</td>
</tr>
<tr>
<td>$b$</td>
<td>width of plate</td>
</tr>
<tr>
<td>$D_{by}, D_{bc}: 0, 1, 2, \ldots$</td>
<td>bending rigidity of beam</td>
</tr>
<tr>
<td>$D_p$: bending rigidity of plate</td>
<td></td>
</tr>
<tr>
<td>$E_p, E_b$: Young’s modulus of plate and the $i$th beam, respectively</td>
<td></td>
</tr>
<tr>
<td>$f$: complex amplitude of the excitation force</td>
<td></td>
</tr>
<tr>
<td>$G_p$: extensional rigidity</td>
<td></td>
</tr>
<tr>
<td>$G_{bi}: 0, 1, 2, \ldots$</td>
<td>shear modulus of the $i$th beam</td>
</tr>
<tr>
<td>$h$: thickness of plate</td>
<td></td>
</tr>
<tr>
<td>$I$: structural intensity along the $x$-axis</td>
<td></td>
</tr>
<tr>
<td>$I_{bi}$: torsional stiffness of the $i$th beam</td>
<td></td>
</tr>
<tr>
<td>$K_p$: stiffness matrix of the plate</td>
<td></td>
</tr>
<tr>
<td>$K_B$: stiffness matrix of the beam</td>
<td></td>
</tr>
<tr>
<td>$K_{bi}$: stiffness matrix due to the coupling between the plate and beams</td>
<td></td>
</tr>
<tr>
<td>$K_{y,b}, K_{s,b}, K_{a,b}, K_{t,b}$, $K_{y,b}, K_{s,b}$, $K_{a,b}, K_{t,b}$</td>
<td>stiffnesses of rotational springs at the endings of the $i$th beam</td>
</tr>
</tbody>
</table>

*Corresponding author (email: w-li@wayne.edu)
$K_{i0}^p, K_{i1}^p$ (or $K_{i0}^p, K_{i1}^p$): stiffnesses for rotational springs at $x = 0$ and $a (y = 0$ and $b)$, respectively

$K_{i3}^{rb}, K_{i1}^{rb}, K_{i2}^{rb}$: stiffnesses for rotational coupling springs between the $i$th beam and plate

$k_{y,i}^b, k_{x,i}^b, k_{z,y}^b, k_{x,z}^b, k_{x,i}^b, k_{z,i}^b$: stiffnesses of linear springs at the endings of the $i$th beam

$k_{y,i}^{rb}, k_{x,i}^{rb}, k_{z,y}^{rb}$: stiffnesses for linear coupling spring between the $i$th beam and plate

$k_{y,i}^{rb}, k_{x,i}^{rb}$: stiffnesses of linear springs at $x = 0$ and $a (y = 0$ and $b)$, respectively

$k_{x,i}^{rb}, k_{y,i}^{rb}$: stiffnesses for the tangential springs at $x = 0$ and $a (y = 0$ and $b)$, respectively

$L$: Lagrangian operator

$L_b$: beam length

$L_c$: length of the $i$th beam

$N_{eb}$: number of the beam stiffeners

$N_{nx}, N_{ny}$: in-plane normal and shear stresses

$l_x, l_y$: directional cosines of a beam

$M, N$: truncation number of the plate Fourier expansion series

$M_{ex}, M_{ey}$: bending moment, torsional moment

$M_b$: mass matrix of the plate

$M_i$: mass matrix of the beams

$N_b$: number of the beam stiffeners

$\langle P_i \rangle$: time-averaged power input into the stiffened plate

$Q$: shear force

$S$: cross-section area of the $i$th beam

$T$: total kinetic energy

$\{T_i\}$: time averaged and steady state kinetic energy

$T_{eb}$: kinetic energy of the $i$th beam

$T_{ps}$: kinetic energy of the plate

$u_{ps}, v_{ps}, w_{ps}$: in-plane and flexural displacements of plate

$u_{ax}, w_{ax}$: axial, flexural displacements of beam

$v$: velocity at the driving location

$U_{ps}, V_{ps}$: coefficient vectors of in-plane and flexural displacements of plate

$U_{ps}, W_{ps}$: coefficient vectors of in-plane and flexural displacements of the $i$th beam

$V$: total potential energy

$V_{ps, out}, V_{ps, in}$: strain energies due to the bending and in-plane motion respectively

$V_{ps, c}$: potential energy stored in boundary springs of the plate

$V_{ps, c}$: strain energies of the $i$th beam

$V_{ps, c}$: potential energy stored in boundary springs of the $i$th beam

$V_{ps, c}$: potential energies due to couplings between the $i$th beam and plate

$\mu$: Poisson’s ratio

$\rho_o, \rho_i$: mass density of the plate and the $i$th beam

$\omega$: frequency in radian

$\theta_b$: torsional displacement of the beam

$\phi_i$: coefficient vector of rotational displacements of the $i$th beam

$\phi$: orientation angle of the beam

$\Omega = (\omega l_i^2 / \pi^2)^{1/2} \sqrt{\rho_i h / D_i}$, frequency parameter

1 Introduction

Engineering structures composed of stiffened plates are extensively used in many fields to minimize the weight while providing sufficient strength. The determination of energy distributions of and power flows through stiffened plates is of significant importance in the study of vibration transmissions and structure-borne sound propagations in built-up structures. Many researchers have studied vibration power flows in structures using various analytical and numerical methods.

The wave propagation approach has been developed primarily to study power flows through connections of semi-infinite or infinite plates and beams. Goyder and White [1, 2] examined the near and far field power flows of an infinite plate with a single line stiffener under a force or moment excitation. When torsional waves were excited in the beam, it was found that the beam played a dominant role at high frequencies and the plate became more important at low frequencies regarding the vibration power transmission. Combining the Bloch theorem with the wave propagation approach, Mead [3] used phased array receptance functions to obtain an analytical solution for a plate stiffened by an infinite number of beams. He also studied the relationship between the wave propagation constants and the “pass/stop bands” of an infinite periodic ribbed plate. Mead’s work has been extended by many researchers to address vibrations and power flows of periodic structures from various aspects [4–7].

Statistical Energy Analysis (SEA) has been used to predict power flows between coupled beams and plates at high frequencies or for high modal density [8, 9]. Limited by its basic assumptions [10] which include, for example, weak coupling, reverberant wave fields, and the “rain on the roof” excitations, SEA can only provide the global space- and frequency-averaged information of field variables at high frequencies without indicating local distributions of the variables [11].

Power flow paths are often identified with the help of structural intensity that indicates both the magnitude and the direction of energy flows at any point on a structure. Finite Element Analysis (FEA) has been extensively adopted to investigate power flows and the structural intensities of connected systems [12–16]. Hambric [12] considered a dis-